A Review of Impedance-Based Fault Locating Experience

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INTRODUCTION

Five years of field experience with fault-locating distance relays demonstrate the value, performance, and practicality of these devices in protective relaying and fault-locating applications. The careful design of signal processing which has made these devices good relays also contributes to their good performance as fault locators even though the requirements for relaying and fault locating are quite different.

This paper reviews fundamental fault-locating principles and field experience, discusses and analyzes special cases, and points out how fault locating has benefitted protection as well as operation of power systems. Several unique applications are also presented.

REVIEW OF TRANSMISSION LINE FAULT-LOCATING TECHNIQUES

Electronic methods for locating transmission line faults include:

1. Relating oscillographic readings to short-circuit study data. This method is slow, requiring the oscillograms to be retrieved, and must be performed by individuals skilled in reading the oscillograms. It also depends on having short-circuit program data for the system configured as it was when the fault occurred. It is very intolerant of fault resistance.

2. Processing digital oscillograph records in a fault-locating program. This method is also slow. Large data records must be retrieved by computer, and then be processed by a person skilled in operation of the program. Skills include selecting the proper voltage and current channels, and selecting the data carefully from the faulted waveforms.

   Neither of the above schemes is of much use to an operator, who must make a decision to sectionalize, reclose, dispatch a crew, etc. soon after an event occurs.

3. Two-end travelling-wave fault locators. These schemes measure the relative time of arrival of the travelling wavefront produced by the fault, at the two ends of the line. The fault location is calculated from this measurement. A high-speed (wide bandwidth) communications channel is required for accurate time measurement. Equipment at both ends of the line, as well as the communications channel, must be operating in order to obtain a measurement. Traditionally, no remote communications of the fault location from the substation to the operator have been available.

4. One-end travelling-wave fault locators. A scheme has been developed for HVDC lines which does not require equipment at both line ends, and which needs no wide-bandwidth communications channel. Further research is necessary before the technique is practical for ac transmission lines.

5. One-end impedance-measuring fault locators. These devices calculate the fault location from the apparent impedance seen looking into the line from one end. They have proven to be the most practical, since no communications channel (other than possibly one for remote reading of the fault location) is required, and they are generally easy to install and operate. Commercial equipment based on analog techniques was not widely accepted, due to marginal performance. Several digital systems have been available for some time: these offer superior performance to the analog predecessors. Indeed, one-end impedance-measuring fault locators are included in several digital distance relay packages, and the feature of fault locating adds little or nothing to the cost of the total system.

6. Two-end impedance-based fault locators. Given the voltage and current information at both ends of the line during a fault, the fault location can be calculated. The advantage of one such scheme, described in Reference 1, is that ground faults can be located without knowing the zero-sequence impedance of the transmission line. The disadvantage of this scheme is similar to locating faults with digital oscillographs: the data must be retrieved and then processed by a relatively skilled individual. Although communications and computer resources could be applied to totally automate two-end schemes, the complexity and loss of availability (communications must be successful to both line ends, and the computer to process the two records must be available) may seldom be worth the performance advantage. Indeed, the techniques discussed for handling sources of error in single-end fault locating bring the performance of single-ended schemes up to par with two-ended schemes in most cases.

PRINCIPLES OF IMPEDANCE-BASED FAULT LOCATING

Locating faults requires many of the same signal processing steps as protecting transmission lines.

To accurately locate all fault types, the phase-to-ground voltages and the currents in each phase must be measured. (However, as we discuss later, when line-to-line voltages only are available, it is possible to locate phase-to-phase faults accurately. Ground faults can also be located reasonably well in most cases, if the zero-sequence source impedance is known.)

The phasor quantities must be extracted, a process which requires filtering to ensure that transients do not affect the measurement of phasor quantities. We have found a combination of analog and digital filtering that is simple and effective. An analog filter removes all high frequency components, and a digital filter removes dc offset.

Knowledge of the fault type is essential for accurate single-end fault locating, as the fault type determines the measuring loop to be used. In the digital distance relay/fault locator equipment of our manufacture, we use two different techniques. One technique is to determine the fault type from the relay elements which operate. The other technique is to use a separate fault-type determination process exclusively for the fault locator. This latter technique tests and compares the phase and residual currents. Another technique which has been used by others is to use the information from external starting elements. (Still another way, which has been used by programs that analyze digital oscillographic records, is manual specification of the fault type, relying on a skilled operator for fault-type determination.)

One of the following impedance calculations may be employed, depending on the fault type, to calculate the apparent positive-sequence impedance to the fault:

Ground Faults:

AG: Z1 = VA/(IA + k * IR) Where k = (Z0 - Z1)/3 * Z1
BG: Z1 = VB/(IB + k * IR)
CG: Z1 = VC/(IC + k * IR)

Phase-to-Phase and Phase-to-Phase to Ground Faults:

AB or ABG: Z1 = VAB/IAB
BC or BCG: Z1 = VBC/IBC
CA or CAG: Z1 = VCA/ICA

Three-Phase Faults:

Any of the above equations.

The measured impedance unfortunately depends on many factors not represented in the equations. These include no or imperfect transposition between the fault and the measurement bus, mutual coupling to nearby circuits, load flow and fault resistance. Other problems arise from taps, conductor configuration changes, instrument transformer errors, nonuniform or unknown soil resistivity, etc. Fortunately, there are ways to handle or discover most of these problems, and many of them are often insignificant, as is explained later.

Once the apparent positive-sequence impedance to the fault is calculated, the distance to fault is determined by dividing the measured reactance by the total reactance for the line and multiplying by the line length. This eliminates the effects of fault resistance under conditions of light loading. On more heavily-loaded lines, faults with considerable resistance are not accurately located by this method, since the voltage drop at the fault in the fault resistance has both a resistive and a reactive component, as seen from either end. The reactive component of this drop is an error term not eliminated by this simple calculation. Takagi et al (Reference 2) provided a simple calculation which takes prefault load flow into account to reduce the effects of fault resistance and load flow on fault location calculations tenfold. Reference 1 works through some examples showing the difference between a straight reactance calculation and the Takagi approach, and a later section of this paper compares the reactance and Takagi methods for a ground fault.

The Takagi approach begins by writing the equation for the voltage at one line end, e.g. the "s" end, in terms of the current measured at the relay location during the fault, the total fault current and the fault resistance:

$$V_S = mZ_1 I_S + R_f I_f$$

Where:

m	=	Per-unit distance to the fault
Z_l	=	Total positive-sequence impedance of the line
I_s	=	Relay current
R_f	=	Fault resistance
I_f	=	Total current in the fault

The total fault current is the sum of the fault current component from the "s" and "r" ends:

$$I_f = I_{fs} + I_{fr}$$

At the "s" end, the fault current component I_{fs} is the difference between the fault and prefault currents:

$$I_{fs} = I_s - I_{s0}$$

To minimize the effects of the fault-resistance term, the equation for V_s is multiplied by the complex conjugate of the fault-current component I_{fs}, and the imaginary parts are saved:

$$Im(V_s I_{fs}^*) = m\ Im(Z_1 I_s I_{fs}^*) + R_f\ Im((I_{fs} + I_{fr})\ I_{fs}^*)$$

The multiplication makes the fault-resistance term nearly real, so its imaginary part is, in general, negligible. Therefore, if we neglect the generally-small imaginary part of the last term, the distance to fault becomes the ratio:

$$m = Im(V_s I_{fs}^*) / Im(Z_1 I_s I_{fs}^*)$$

The Takagi paper (Reference 2) uses the alpha component of the fault current component for I_{fs}, since it is more uniform from line end to line end, and less affected by system grounding differences.

Reference 1 describes a two-ended algorithm. It uses the phasor information from both line ends to determine the fault location. The advantage of this scheme is that it does not need to depend on knowing the zero-sequence impedance of the line, a parameter which depends on the soil resistivity among other things. The scheme is also free of effects of zero-sequence mutual coupling. The two-ended algorithm recognizes that the voltage along the line from either end can be represented as a function of the distance to fault. If there is only one fault, then the equations for voltage computed from either end can be equated, and the distance to fault solved for as follows:

$$V_f = V_s - mZI_s$$
$$V_f = V_r - (1-m)ZI_r$$

Solving for m:

$$m = (V_s - V_r + ZI_r)/(Z(I_s + I_r))$$

Reference 3 gives an example of the application of this method to a 345 kV line.

MANAGING SOURCES OF ERROR IN IMPEDANCE-BASED FAULT LOCATING

Fault Resistance and Load Flow

The combination of these two factors introduces errors, which are serious in straight reactance calculations, and minimized by the Takagi algorithm.

On radial lines or any line where the infeed from the remote end is small compared to the total fault current, or when the load flow is small on an interconnection, the fault-locating errors due to fault resistance and load flow are negligible, even with a straight reactance calculation. This is useful to know, since a hand calculation of fault location using the reactance calculation is easier than a hand calculation using the Takagi algorithm. Indeed, when prefault information is not available, the Takagi algorithm cannot be used, and we are left with the reactance calculation.

Figure 2 shows a two-source equivalent system, with buses S and R. The EMF behind bus S leads that behind bus R by the power angle delta. To see the effects of fault resistance and load flow on the reactance and Takagi calculations, a ground fault, having resistance the same as the positive-sequence impedance of the line from S to R, was applied at the midpoint of the transmission line. Five test cases were produced using different power angle values, and with equal Thevenin impedances behind the buses. The system was homogeneous for these five cases.

As the theory predicts, the Takagi algorithm is unaffected by the fault resistance and the load flow. The table included with Figure 2 shows this, with per-unit distance to fault values of 0.5 from both ends for all five cases.

Base Case:

$Z_{S1} = Z_{R1} = 2 \angle 80°$
$Z_{S0} = Z_{R0} = 3Z_{S1}$
$Z_{L1} = 8 \angle 80°$
$Z_{L0} = 3Z_{L1}$

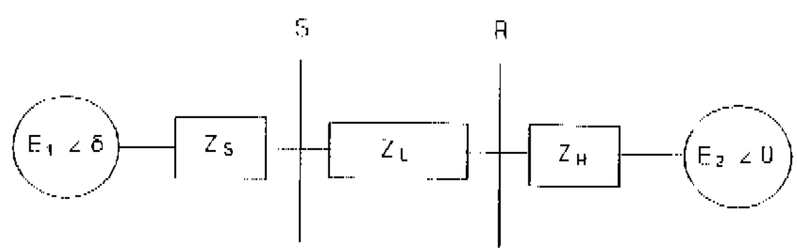


Figure 1: Ground faults at line midpoint (m = 0.5); R_F = 8

Test Cases:

	δ	mxs	mxr	mts	mtr
1.	0	0.5	0.5	0.5	0.5
2.	+15°	0.362	0.812	0.5	0.5
3.	+30°	0.305	1.425	0.5	0.5
4.	+45°	0.286	2.109	0.5	0.5
5.	+60°	0.286	2.127	0.5	0.5
	$\angle Z_{R1} = 60°$		$\angle Z_{R0} = 80°$		
6.	0°	0.528	0.507	0.547	0.507
7.	15°	0.369	0.831	0.539	0.509
	$Z_{R1} = 1 \angle 90°$		$Z_{R0} = 1 \angle 90°$		
8.	0°	0.482	0.524	0.478	0.523
9.	15°	0.826	0.787	0.483	0.530

mxs, mxr: Per unit distance from S or R indicated by the reactance algorithm.

mts, mtr: Per unit distance from S or R indicated by the Takagi algorithm.

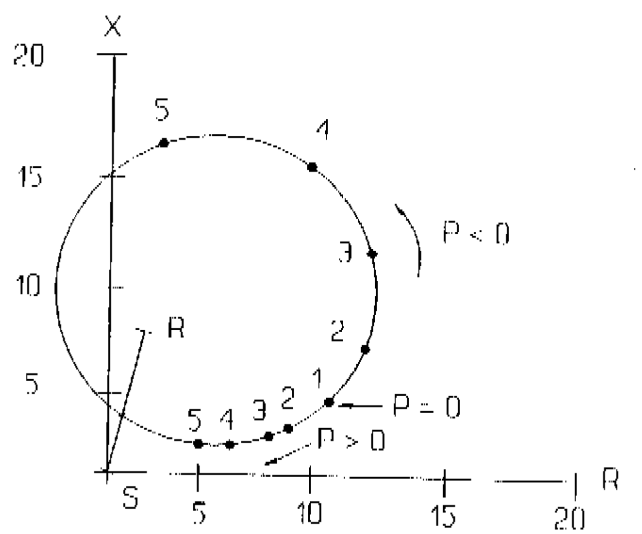


Figure 2: **Apparent Positive-Sequence Reactance Locus for a Ground Fault Midway Between S and R**

The reactance algorithm is strongly affected by load flow. When E1 leads E2 by 15° (Case 1), corresponding to left-to-right power flow, the fault appears much closer to S and farther from R. The apparent positive-sequence impedance is plotted in the figure, and we see that its locus is a circle, passing through the actual positive-sequence reactance values for the fault at two points, corresponding to power angles of 0° and 180°.

Two cases (6 and 7) show the effect of changing the angle of the source impedance at one end (R). The positive-sequence source impedance was changed from 80° to 60°. The reactance and Takagi algorithms are affected at both ends. Indeed, for a power angle of zero, the Takagi result at R is actually worse than the reactance value (0.547 vs 0.528)! Once power begins to flow, the reactance algorithm shows serious degradation similar to the first five cases, but the Takagi algorithm is much better behaved.

In Cases 8 and 9, we set the positive and zero-sequence source impedances at R equal to each other, as might be the case for a grounded transformer bank. Again, when the power angle is zero, the reactance and Takagi algorithms are both affected in similar amounts. However, power flow favors the Takagi algorithm.

Zero-Sequence Impedance Errors

An examination of Carson's equations shows that the zero-sequence impedance of a transmission line depends on the soil resistivity, as well as the conductor size, configuration, and height. Since the soil resistivity is not known precisely, and is not constant, Z0 is never precisely known. Clearly, the positive-sequence impedance-to-fault calculation for ground faults, given earlier, depends on the k factor, which, in turn, depends on Z0. As an example for a 345 kV line, if the actual Z0 is 20% lower than the value used in the fault locator equation (or in a distance relay setting), then the fault locator will indicate about 15% short (or a distance relay would overreach its setting by 15%).

When the fault location is known and the fault resistance is low, Z0 can be solved for. Z1 is assumed known accurately, and the k factor is found using the impedance-to-fault equation given earlier, with the faulted phase voltage, the faulted phase current, and the residual currents as inputs. Then, Z0 is found from the k factor and Z1.

Because the setting and coordination of ground distance and overcurrent relays depend so heavily on Z0, verifying the zero-sequence impedance is useful in improving the protection, as well as improving fault locating. References 3 and 4 both discuss cases where substantial zero-sequence impedance errors were discovered, and where improved Z0 values were calculated from the data saved by fault-locating relays.

Zero-Sequence Mutual Coupling

On well-transposed lines, mutual coupling from parallel circuits is significant only in the zero-sequence network. Accurate fault locating from one line end theoretically requires knowing the voltage at the line end, the currents there, and the currents of any other circuits having significant mutual coupling to the monitored circuit.

On a parallel line application, where Z0M, the mutual zero-sequence impedance, is uniform along the double circuit, it is possible to modify the fault locating equation to include the effects of currents in the parallel unfaulted circuit. This requires another measurement, namely the residual current in the parallel circuit. Relays and fault locators have been made which take advantage of this.

More often than not, the problem is more difficult. The mutual coupling may not be uniform, if conductor size changes or the line configuration changes along the line. Even worse, circuits may parallel the monitored line for only part of the way, and not bus into the same substation where the monitored circuit is. Figure 3 shows some difficult situations. Thus, there are many cases where there is not enough information at any one substation to make an accurate calculation of fault location without communications or other help.

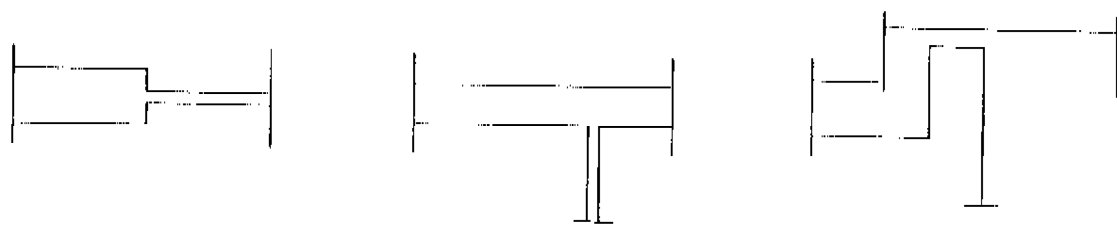


Figure 3: Difficult System Arrangements for Zero-Sequence Compensation

Several approaches are available to attack these problems:

1. Calculate the effects of possible mutual-coupling errors and explain them to the users of fault locating information if the errors are significant.

2. Calculate the effects, and make a correction chart or charts. One way to do this is to use a short circuit program to generate short circuit data for faults along the line of interest, and to present these voltages and currents to a fault locator with the help of a test set. Then the indications can be plotted against the actual locations, resulting in a correction chart. This need only be done for ground faults, as the zero-sequence mutual coupling does not affect phase faults. An analytic way is to use the short-circuit study data as input to the apparent-impedance equations given earlier, and then construct a nomograph.

3. Use a fault locator with an input for the residual current from the offending circuit. This is a simple solution if the parallel circuit is uniformly coupled and the current signal is available.

4. Use the two-ended algorithm. It does not depend on zero-sequence self or mutual impedance. It does require that the data from both ends of the line be brought together and processed, a procedure that is not very convenient or fast, from an operations point of view.

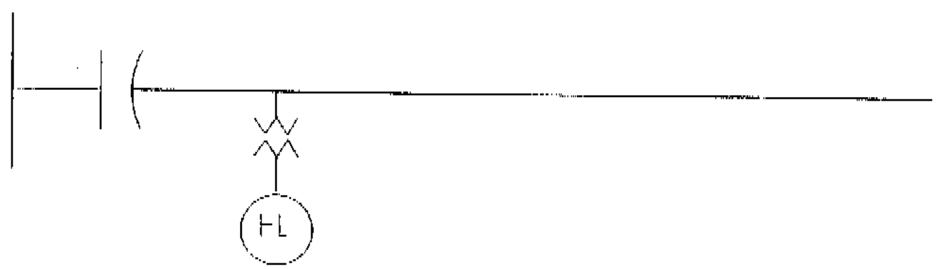
5. Completely represent the power system in the area of interest, and bring together measurements from all points in the system affecting the measurement. This "total solution" approach is possible with today's communications and data recording equipment, but the speed, modelling (setting), expense, and complexity are probably not justified.

Series Compensation

Series capacitor compensation offers steady-state as well as transient error problems.

Figure 4 shows two different fault locating problems, where series capacitors are applied. In 4a, the desired line-side potentials are available, and no steady-state problem exists. These potentials may be used with the phase currents as always.

a) Line-Side Voltages



b) Bus-Side Voltages

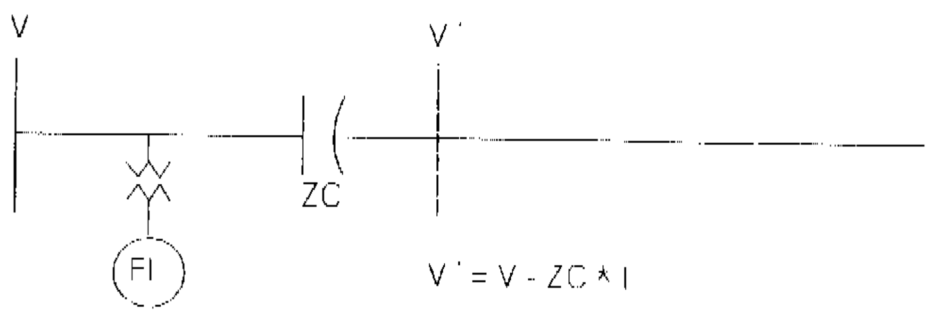


Figure 4: Series Compensated Line Considerations

In 4b, a steady-state source of error is possible, because the voltages measured by the bus potential transformers include the line drops to the fault and the potential differences across the capacitors. If protective gaps flash, shorting out the capacitors, then the steady-state error vanishes. If they do not flash, then the voltage drop must be compensated for. This can be done analytically, as follows. Consider a fictitious bus on the line side of the capacitors. Calculate the voltage at this bus, using the measured phase currents, voltages, and the impedance of the capacitors. That is, for each phase:

$$V' = V - ZC * I$$

Reference 4 describes a test of this approach on a 500 kV line. Practical problems for bus voltages include knowing whether or not the gaps flashed, and how much compensation was in service at the time of the fault. The presence of the series capacitors presents a serious transient problem, unless the protection around the capacitors operates. The transients may have signal components at frequencies below 60 Hz, making them difficult to filter out. If the fault duration is several cycles, then averaging several fault location calculations, made with well-filtered data, does a reasonable job of reducing the effect of the transients to acceptable levels.

Fortunately, the protection around the capacitors operates for many faults, especially close-in ones, so that the performance of impedance-based schemes is acceptable without much additional consideration. If fault locators are used at both line ends, then their results can be compared for validity.

Conductor Size and Configuration Changes

Often, especially at lower voltages, Z0 and Z1 may change along the line, due to changes in the size or configuration of the conductors. The following calculation procedure for locating a phase fault on such a line could be used:

1. Calculate the apparent positive-sequence impedance to the fault.

2a. If it is less than Z1 for the first line section, then determine the fault location as usual.

2b. If it is greater than Z1 for the first line section, then subtract the impedance for that line section, and compare the result against the impedance of the next line section. If the difference is less than the second section impedance, then the fault location is the sum of the length of the first section and the fraction of the length of the second section determined by the usual procedure. Continue the procedure if the difference is greater than the impedance of the second section, by subtracting out the impedance of the second section, etc.

Ground faults are more complicated, because Z0 and Z1 may not stay in constant proportion, making the k factor depend on the line section.

The following calculation procedure for locating a ground fault on such a line could be used:

1. Calculate the apparent impedance, using the k factor for the first section, if it is a ground fault. If the impedance is less than that of the first section, then the fault location can be computed. If it is greater, then create a fictitious bus at the end of the first section. Compute the voltage there as follows (for an AG fault):

$$VA' = VA - Z1 * (IA + k * IR),$$

where VA' is the fictitious bus voltage, VA is the measured voltage, Z1 is the positive-sequence impedance of the first line section, k is the k factor for the first line section, and IA and IR are the faulted phase and residual currents.

2. Use VA', IA, IR and k' (the k factor for the next line section) to calculate the apparent positive-sequence impedance from the fictitious bus to the fault or possibly into the third section.

3. If the fault is beyond the second section, then compute the voltage at the boundary between the second and third sections as follows:

$$VA'' = VA' - Z1' * (IA + k' * IR),$$

and find the apparent positive-sequence impedance into the third section, etc.

A <u>much</u> easier approach is to analyze the effects of the conductor changes. Often the size/spacing changes make insignificant changes to the line constants. If the changes are significant, then the short-circuit calculation procedures discussed earlier can be used to make nomographs for interpreting the readings of a standard fault locator.

Tapped Load

Tapped load seldom makes any significant difference in the fault location, since delta-wye connections of the transformers are usually used (no ground source), and since the impedances of the transformers are generally large compared to the line impedances. It does make a difference when the load current is near the short-circuit current, however.

One situation of practical interest having a rather simple theoretical solution is the case of tapped loads on radial circuits. Appendix I describes a solution which works as long as the tapped loads do not affect the zero-sequence network.

Three-Terminal Lines

Single-end fault locators indicate accurately up to the tap point. Beyond that point, infeed causes an excessive distance to fault indication.

The simplest way to handle a three-terminal line when substantial sources exist at each of the three terminals is to install a fault locator at each of the three terminals. When a fault occurs, obtain the readings from all three units, and accept the reading from the terminal showing the fault location short of the tap point.

Cogeneration

If a cogenerator is connected to the system at a tap point, then the fault locator indicates accurately up to the point of the tap, and indicates long beyond that point due to the infeed of unmeasured current from the cogenerator. If there are sources at both ends of the line, then a unit at each end of the line can be used, and the measurement short of the cogenerator tap is accepted as the more accurate indication.

Another approach is to only consider permanent faults. When a fault occurs, the cogenerator must also trip. If the utility reclose attempt results in a second (e.g. permanent) fault, then the fault location will be accurate, since the cogenerator would not be connected at the time of the reclose. Of course, an analysis of the amount of infeed the cogenerator can provide, compared to the utility source of fault current, may reveal that the cogenerator infeed can be neglected in some cases.

Short-Duration Faults

Impedance-based fault-locating techniques require that the system-frequency voltage and current quantities can be accurately measured. This requires filtering already mentioned, and signals of long enough duration to measure.

A two-cycle fault is probably the practical lower limit on fault duration for consistently reasonable results.

Travelling-wave relays and very fast breakers can provide clearing times of less than one cycle. Surge arresters may also operate fast enough to produce fault durations of less than one cycle.

For such short duration faults, the current never reaches its faulted steady state, and the voltage never drops to its faulted steady state, so the tendency is for the fault location to indicate long. Travelling-wave approaches may be the only solutions.

CCVT Transients

Capacitively-coupled voltage transformers delay the secondary voltages and create transients. Filtering in the fault locator must remove transient components created by the CCVTs. In the SEL equipment, a net band-pass filtering process is provided by an analog low-pass filter with a cutoff of 84 Hz, and by a digital filter which rejects DC. This combination has proven itself effective, as long as the fault duration is long enough to allow the 60 Hz component of the voltage to reach a steady state. The time this takes depends on the CCVT design.

CT Saturation

CT saturation robs the fault locator of current, making the measured impedance increase above its expected value without saturation, and the fault location indicates long. Fortunately, CT saturation is only likely to happen for very close-in faults, where the voltage is small. Although the percentage error in the distance calculation might be large, the absolute distance error probably will not be. For example, if CT saturation causes a 20% error in the current measurement, and the fault is only one mile away from the station, then a distance error of 0.2 miles would result, which is tolerable in a practical sense. If the fault is two miles away, the CT saturation error would probably drop to less than 10% in this case, corresponding to a distance error of less than 0.1 mile.

Delta-Connected PTs

At the outset, we assumed that phase-to-neutral voltages are available for all three phases. In some applications, only phase-to-phase voltages are available. Such is the case where two line-to-line voltage transformers are used instead of three phase-to-ground transformers.

Measurement of the line-to-line voltages denies the fault locator of any direct knowledge of the zero-sequence voltage component at the point of measurement, making ground-fault location more difficult. Phase-fault locating is not affected, since phase faults do not produce zero-sequence voltage.

Reference 5 describes a fault-locating relay which estimates the faulted phase-to-neutral voltage from the phase-to-phase voltage measurements and from an estimate of the zero-sequence bus voltage calculated from the zero-sequence current and a setting for the zero-sequence source impedance.

For an AG fault, the voltage VA can be calculated from the line-to-line voltages and the zero-sequence current using the following theory:

$$\begin{aligned} VA &= V1 + V2 + V0 \\ &= 1/3[VAB - VCA] + V0 \end{aligned}$$

Estimate V0 using V0 = - Z0 * I0, where Z0 is the zero-sequence source impedance, and I0 is the zero-sequence current.

Then,

$$VA = 1/3[VAB - VCA] - Z0 * I0$$

Since VAB, VCA and I0 are available from the PTs and the CTs, all that is newly required is Z0, the zero-sequence source impedance.

This scheme has been in use for about three years, with good results. One theoretical limitation, which has not proven to be a practical limitation, is that the zero-sequence source impedance depends on the fault location, in a looped system.

Transmission Line Asymmetry

Does the fault location calculated by an impedance method vary from phase to phase, because of the asymmetrical construction of the line? To address this question, we analyzed two line configurations in Appendix II. One is a horizontal line configuration; the other is vertical. The following observations summarize the results of that study.

1. In the horizontal configuration, there is virtually no ground-fault phase dependency on accuracy, as long as the residual current and the faulted phase current are close together. (This condition minimizes the effects of the very different mutual impedances between the center phase and an outside phase, versus the coupling between the outside phases.) There is substantial phase-fault dependency; however. For the example configuration of Appendix II, a phase fault between two adjacent phases will indicate short, by several percent. A phase fault between the outside phases will indicate long.

2. In the vertical configuration, there is some ground-fault phase dependency, due to the differences between the self impedances, owing to the different conductor heights over ground. For the example, the dependency was about 0.3%, small enough to be neglected in most applications. The phase fault phase dependency is similar to the horizontal case.

3. The unequal mutual terms in the phase impedance matrices lead to load-current dependencies which affect fault locating. If the load current is similar to the fault current, then the errors due to load current can be several percent.

VALUE OF FAULT LOCATING TO PROTECTIVE RELAYING

Knowing the location of faults, especially permanent ones, is of obvious benefit in operating power systems. Several years of experience with fault-locating relays show that fault locating is also very valuable in the analysis of protective relaying schemes. Several examples follow.

1. A phase-to-phase fault occurred one substation away from a fault-locating relay: the distance-to-fault indication was incorrect. This led to the discovery that the positive-sequence impedance for the transmission line had been calculated wrong. (The wrong conductor spacing had been entered into the line constants program years before the application of the fault-locating relay.) The discovery was important in correcting the settings of the line protection.

2. When fault-locating relays are used at both line ends, or when a fault occurs at a known location, the line constants can be checked. In several cases, short distance-to-fault indications have revealed errors in the zero-sequence impedance for the line, most likely due to high estimates of the soil resistivity.

3. A fault on the first 80% of the line normally should operate the appropriate Zone 1 distance element. However, if enough fault resistance is present, the apparent impedance to the fault may be outside the Zone 1 circle, but still inside Zone 2 or 3. The fault locator indicates the distance to fault, not the apparent impedance, so it is easy to sort out faults in Zone 1 distance, which do not pick up the Zone 1 element, from faults beyond Zone 1 distance. A similar argument exists for better understanding high-resistance faults detected by ground-overcurrent elements.

4. In a trip-reclose-trip-lockout sequence, usually (but not always) the same fault is responsible for both trips. Fault locating makes it easy to determine if two faults or one fault actually occurred. For example, the first fault may be due to a tree branch. On reclose, say, with the remote end open, a weak insulator string could flash over on a transient overvoltage, at a location and phase different from the first fault. The fault locator must be able to respond to faults in rapid succession to be useful in this way. (The fault-locating relays of our manufacture save up to 12 events triggered in rapid succession, and satisfy this requirement.)

In the future, fault location may become a valuable input in reclosing schemes and automatic sectionalizing schemes.

TWO PRACTICAL EXAMPLES

Rochester Gas and Electric Multiple Circuit Application

Rochester Gas and Electric Company engineers have applied fault-locating relays at three installations, each of which uses a single digital fault-locating distance relay to locate faults on several 34.5 kV lines.

One installation (Station 216) uses the low side transformer current transformers, which provide the total current in four 34.5 kV circuits.

Another installation (Station 204) sums the currents from two transformers and an additional tie line to provide the fault-locating relay with the sum current into a total of six 34.5 kV circuits. RG & E developed nomographs for each of the individual circuits, since the circuits are different and inhomogeneous. One such nomograph is given in Figure 5, for Circuit 759 at Station 204.

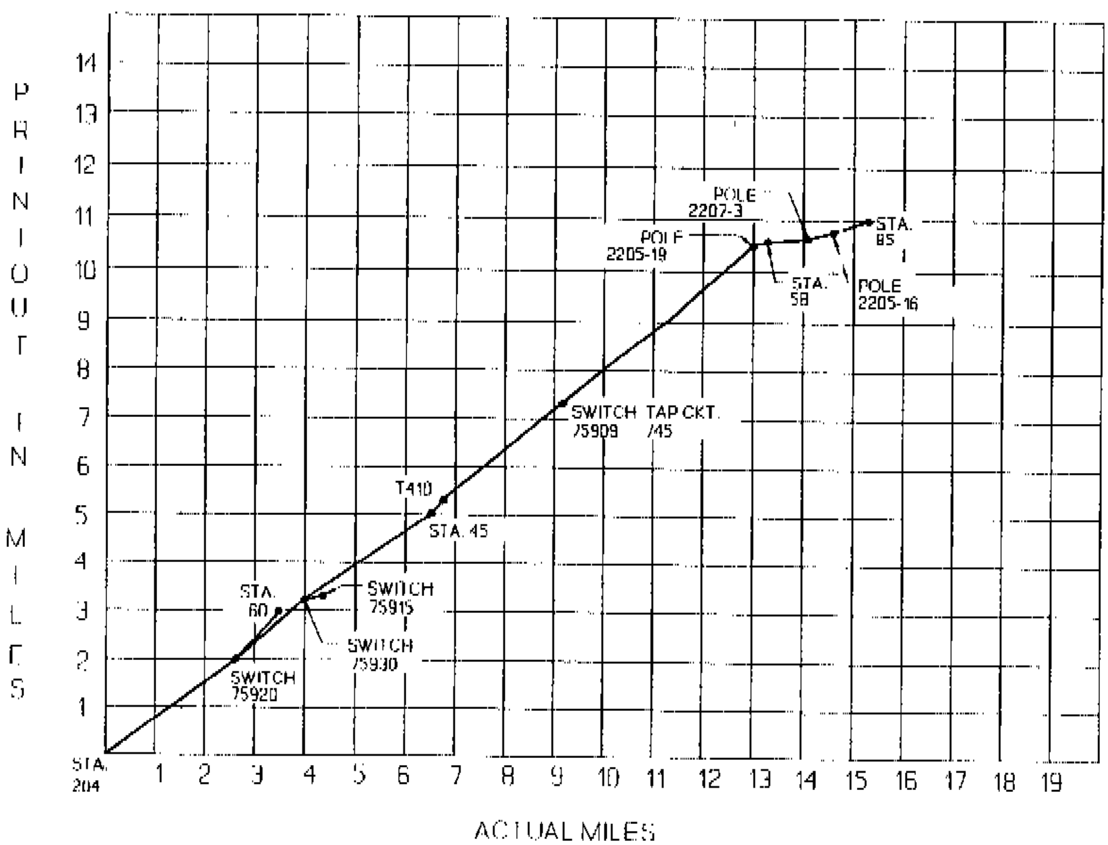


Figure 5: Nomograph - 34.5 kV Circuit 759 - Station 204

A third installation (Station 56) also sums the currents from two transformers and a tie line, to provide fault locating for four circuits using a single fault-locating relay.

Since these are radial feeders, the currents in the unfaulted circuits are load currents. Experience has shown that the load currents usually have little effect on the fault locating accuracy, and Line Operations personnel have developed confidence in the indications. For example, when a tree fault occurred on a line fed by Station 204, Operations obtained the fault location by interrogating it remotely, and then dispatching line crews to isolate the faulted section. The unfaulted sections were restored to service by connecting them to adjacent circuits, and service was restored in 33 minutes.

Sharing one fault-locating relay amongst four to six radial circuits provides very economical fault locating at voltages where fault locating could never be considered before. When load currents are important, they may be handled by the method of Appendix I.

Otter Tail Power Company Impedance Study

The reactance and Takagi fault locating algorithms used by the fault-locating relays depend on accurate values of the positive and zero-sequence impedances to determine locations of faults on the transmission line. Proper operation of the line protection also depends on these impedances.

The positive- and zero-sequence impedances of the transmission line can be verified when a fault locating relay is installed at each end of the transmission line. The positive-sequence impedance can be checked when the location of the fault is known. Once the positive-sequence impedance has been verified it can be used to check the value of the zero-sequence impedance of the line as used by each relay.

A permanent fault occurred on the Jamestown-Center 345 kV line in North Dakota on May 10, 1985, when fault-locating relays were in service at both ends of the line. The permanent fault was located 110.6 miles from the Center substation. The fault was the result of a tornado passing through the line causing two structures to be damaged.

With the data from the fault-locating relays and the known location of the fault, checks can be made on the positive-sequence impedance of the transmission line by calculating the distance to the fault and comparing the calculated distance with the actual distance to the fault.

Reference 2 describes a method to accomplish this comparison, and Reference 3 gives the details of the calculation procedure. A short summary follows:

The distance to the fault from one end of the line can be verified using the positive-sequence impedance of the transmission line using the two-end fault-locating equations given in Reference 2

Let: V_F = The positive-sequence voltage at the fault

$V_{1C,J}$ = The positive-sequence voltages at Center and Jamestown respectively

$I_{1C,J}$ = The positive-sequence current at Center and Jamestown respectively

m = The per unit distance from Center to the fault. (i.e., fault at Center m=0, fault at Jamestown m=1)

Z_{1L} = The positive-sequence impedance of the transmission line

The two equations for the positive-sequence voltage at the fault (one written using the data from each end) can be set equal to each other and solved for the distance m:

$$V_{1C} - I_{1C}(mZ_{1L}) = V_{1J} - I_{1J}[(1-m)Z_{1L}]$$

$$m = \frac{V_{1C} - V_{1J} + I_{1J}\, Z_{1L}}{Z_{1C}\,(I_{1C} + I_{1J})}$$

Positive-sequence quantities from each end of the line, calculated from the event report data saved by the fault-locating relays, are entered into this equation. The resultant value of m, which is a complex number, for the May 10 fault is 0.9112 -7.3°. Using this magnitude of m gives a distance to the fault from Center of 110.7 mi. This value compares very favorably with the actual value of 110.6 miles. This calculation shows that confidence can be placed in the positive-sequence impedance of the transmission line.

With the value of the positive-sequence impedance known, the zero-sequence impedance of the line as used by each relay to locate the fault can be calculated using the data from only one end of the line. The results of these calculations yield impedances which can be directly entered into the relay settings.

When measuring the distance to the fault from one end of the transmission line, all sequence components must be taken into account. If the positive-sequence impedance of the transmission line is equal to the negative-sequence impedance (i.e., $Z_{1L}=Z_{2L}$) and the resistance of the fault is near zero then an impedance to the fault can be calculated using the voltage and current measurements from one end of the line.

An apparent impedance to the fault can be calculated from the faulted phase voltage and a compensated phase current which allows for the impedance of the zero-sequence network to the fault:

$$mZ_{1L} = \frac{V_F}{I_F + KI_0}$$

Where:

m = The per unit distance from the relay to the fault

Z_{1L} = Positive-sequence impedance of the transmission line

V_F = The faulted phase to neutral voltage at the relay

I_F = The faulted phase current at the relay

I_0 = The zero-sequence current at the relay

The compensation factor K is defined by the following equation:

$$K = \frac{Z_{0L} - Z_{1L}}{Z_{1L}}$$

When the fault location and positive-sequence impedance of the transmission line is known, these two equations can be combined and solved for m Z_{0L} as follows:

$$m\ Z_{0L} = \frac{V_F - m\ Z_{1L}(I_F - I_0)}{I_0}$$

Utilizing currents and voltages from Center and a value of m=0.91, the apparent zero-sequence impedance of the transmission line as seen by the Center relay becomes:

Z_{0L} from Center = 71.7 + j206.3 ohms primary

The same procedure is performed with the Jamestown quantities using a value of m=0.09. This yields an apparent zero-sequence impedance of:

Z_{0L} from Jamestown = 52.2 + j205.6 ohms primary

These two line impedances will not necessarily be equal. The differences between these two numbers can be attributed to items such as changes in soil resistivity along the line as well as errors associated with sampling devices supplying quantities to the relays. The latter is especially true in this case where different types of potential devices at Center (CCVT's) and Jamestown (PT's) are used.

These new values for Z_{0L} are substantially lower than the design value of 65.70 + j254.59 from the line constants program. This fact seems to indicate that the actual zero-sequence impedance of the transmission line is less than anticipated. Since these new values are lower, other relay systems utilizing the design value of Z_{0L} will be adversely affected. These new values can be used by the fault locating relay at each end to more accurately determine the location of the transient faults where very little physical evidence of the fault remains to determine the exact location.

Using these new values of zero-sequence impedances, new compensation factors can be calculated and applied to other faults previously recorded. Table 1 shows fault locations calculated by the relay located at each substation using previously estimated values of Z_{0L}.

Table 1: Fault Locations Calculated by Relay with Old Z_0 Values					
Date	Fault Type	Center	Jamestown	Total	% of Line Length
04/08/85	B-G	27.36 mi*	81.28 mi*	108.64 mi	89.4%
04/19/85	C-G	14.51 mi	112.77 mi	127.28 mi	104.7%
04/19/85	C-G	26.19 mi	99.17 mi	125.36 mi	103.1%
05/10/85	B-G	123.75 mi	11.76 mi	135.51 mi	111.5 %
05/30/85	C-G	50.48 mi	78.74 mi	129.22 mi	106.3%
06/25/85	C-G	122.63 mi	4.27 mi	126.90 mi	104.4%

***Note:** These mileages used Z_{0L} = 65.70 + j254.59 ohms primary. Others used Z_{0L} = 55.48 + j215.0 ohms primary.

The fault locations shown in Table 2 below were developed using the re-calculated values of Z_{0L} for each respective relay.

Table 2: Fault Locations Calculated with Corrected Z_0					
Date	Fault Type	Center	Jamestown	Total	% of Line Length
04/08/85	B-G	30.42 mi	93.91 mi	124.33 mi	102.3%
04/19/85	C-G	13.39 mi	112.83 mi	126.22 mi	103.9%
04/19/85	C-G	25.28 mi	98.11 mi	123.39 mi	101.5%
05/10/85	B-G	110.69 mi	10.97 mi	121.66 mi	100.1%
05/30/85	C-G	48.35 mi	77.93 mi	126.28 mi	103.9%
06/25/85	C-G	115.99 mi	3.68 mi	119.67 mi	98.5%

As can be seen from the results, the new measurements from Table 2 are more accurate in terms of the total line length than those measured by the relay without the modified zero-sequence impedance. This is particularly true when comparing the results for the fault on April 8, 1985, where the total line length went from 108.64 miles to 124.33 miles.

The corrected value of Z_0 should be used in adjusting the settings of the distance relays at Center and Jamestown, since the value from the line constants program could result in significant overreaching, as explained below. A residual-current compensated ground-distance relay measures the positive-sequence impedance to the fault, by comparing the faulted phase-to-neutral voltage with a resultant current, which is the faulted phase current plus a constant times the residual current. In equation form, for an A-G fault, this is:

$$Z_1 = \frac{V_A}{I_A + K * I_R} = \text{The positive-sequence impedance to the fault}$$

Where:

$$K = 1/3\ (Z_0/Z_1 - 1)$$

Since K depends on Z_0, an error in Z_0 causes an error in the resultant current.

If we assume that the current distribution factors for the positive, negative and zero-sequence networks are the same, then IA = IR, and we can rewrite the above equation as:

$$Z_1 = \frac{VA}{(1 + K) * I_A}$$

For the same fault, but different values of K, the ratio of reaches is:

$$\frac{1 + K}{1 + K'}$$

Using the calculated and measured values of Z_0 given earlier, this ratio is 1.14, meaning that ground distance relays set with the calculated value of Z_0 would reach 14% farther than their setting. For example, for a Zone 1 setting of 85%, the overreach is 12%, for a total reach of 97%, leaving little margin for other possible sources of error.

Although this example considers distance relay settings, similar conditions hold for the effects of the Z_0 error on the setting of residual overcurrent relays, especially the instantaneous elements.

CONCLUSIONS

Fault-locating relays have demonstrated their ability to do much more than provide protection and fault locating. As the last example shows, for instance, their data have been used to verify line constants.

Many simple techniques have been applied to overcome sources of error in fault locating. Correction tables and charts, used to compensate for conductor changes, etc. are examples.

On radial systems, one fault-locating relay has been applied to cover several feeders, making fault locating available at low voltages, where it was unaffordable before the advent of fault-locating relays.

These contributions to the operation of the electric power system stem from the need in utilities to solve practical problems, the creativity of utility engineers applied to those problems, and the availability of fault-locating relays.

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2. T. Takagi, Y. Yamakoshi, M. Yamaura, R. Kondow, T. Matsushima, "Development of a New Type Fault Locator Using the One-Terminal Voltage and Current Data", IEEE Transactions on Power Apparatus and Systems, Vol. PAS-101, No. 8, August, 1982.

3. R. C. Bartz, E. O. Schweitzer, III, "Field Experience with Fault Locating Relays", 13th Annual Western Protective Relay Conference, October 20-23, 1986, Spokane, Washington.

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APPENDIX I - USING ONE FAULT-LOCATING RELAY ON A RADIAL CIRCUIT WHERE SHUNT LOAD CURRENT IS PRESENT

General Description

One fault-locating relay can be used to locate ground faults on several radial distribution circuits, by using the total current into the group of feeders.

Load currents on the faulted feeder and the other feeders in that group are included in the measured currents. When the load currents are appreciable, errors result. This note shows a simple way to minimize the effects of load currents when locating faults on radial feeders.

Background

The sequence network drawing on the following page is for a radial network with a ground fault on the reference phase.

The positive- and negative-sequence networks include shunt branches at the bus, which represent the net load effects on all of the feeders. No shunt branch is included in the zero-sequence network, as the common assumption is made that the load stations are not zero-sequence sources to the feeders.

Arrows next to the source impedances indicate the assumed point of measurement of the currents. I1 and I2 include the load current. I0 does not. I1 and I2 are not the same as I1F and I2F, respectively. However, it is clear from the drawing that:

$$I0F = I1F = I2F = 1/3\ IR = 1/3\ IAF. \qquad (1)$$

Only I0F is observable under the assumptions made here.

Normally we would calculate the positive-sequence impedance to the fault using:

$$Z1 = VA/(IA + k * IR), \quad \text{where } k = 1/3(Z0/Z1 - 1) \qquad (2)$$

Because of the load currents being measured during the fault, which do not flow in all of the faulted line, if at all, IA is not accurate. In our picture, IA is not equal to IAF. The solution is to use the residual current for phase A, as suggested by equation (1), as follows:

$$Z1 = VA/(IR + k * IR) = VA/(IR*(1 + k)) \qquad (3)$$

Practical Considerations

Data from an event report generated by a fault is easily processed by hand using equation (3), or a short program can be written for that purpose.

A fault-locating relay could be connected so that IA=IB=IC=IR, and then the calculation would be performed directly in the relay; however, this destroys the phase information, which may be useful in identifying trouble.

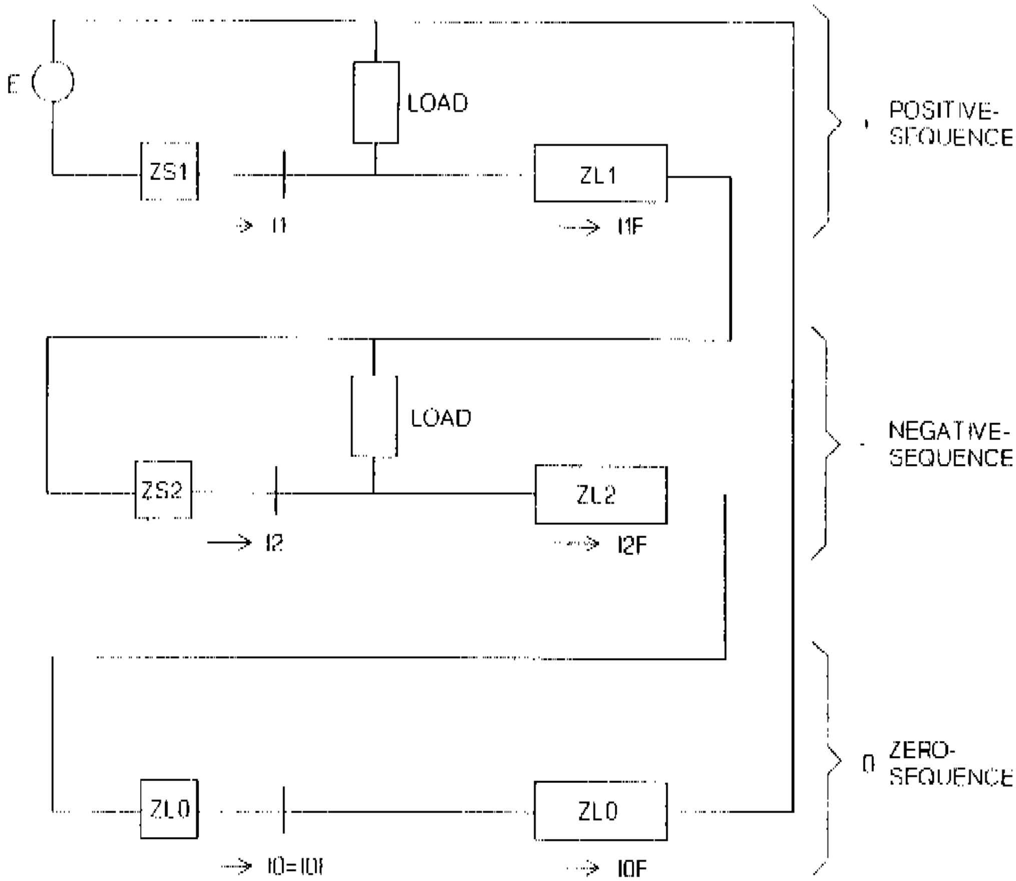


Figure 5: Sequence Network Drawing for Radial System with Tapped Load

APPENDIX II - COMPARISON OF IMPEDANCE MATRICES FOR TWO 100-MILE LINE CONFIGURATIONS

The purpose of this Appendix is to illustrate the dependence of fault locating on the construction of the transmission line. Since it is not possible to build a transmission line where the phase conductors are symmetrically located with respect to each other and with respect to earth and possible ground wires, neither the self impedances (Zaa, Zbb, and Zcc) nor the mutual impedances (Zab, Zbc, Zca, Zba, and Zac) are the same. Because of this, the transformation of the phase impedance matrix to the symmetrical component domain does not yield a diagonalized symmetrical component impedance matrix.

Therefore, the sequence networks, which we normally assume are independent, are in fact not, but are coupled by mutual impedances between them.

For two examples, consider a transmission line with a horizontal (flat) conductor configuration, and with two shield wires. Also consider for comparison a line with a nearly vertical conductor configuration, and a single shield wire. A 10 ohm-meter value of earth resistivity is used for both.

A line constants program was used to find the self and mutual resistances and inductances for the two transmission lines. The matrices are ZH and ZV for the horizontal and vertical configurations.

Average values of the self and mutual impedances are calculated, and the symmetrical component impedance matrices are found by transformations.

Begin by defining the system frequency, a unit conversion relationship between radians and degrees, and the 120° operator.

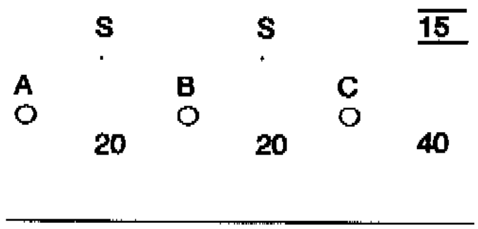
$w := 377 \qquad rad \equiv 1 \qquad a := 1 \cdot e^{j \cdot 120 \cdot deg} \qquad a = -0.5 + 0.866i$

$$degree \equiv \frac{\pi}{180} \cdot rad$$

Let the matrix A be the relationship between the phase voltages (Va, Vb, and Vc) and the symmetrical component sequence voltages (V0, V1, and V2). Its inverse (matrix AS) returns phase quantities from sequence quantities.

$$A := \tfrac{1}{3} \cdot \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \qquad AS := A^{-1} \qquad I := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

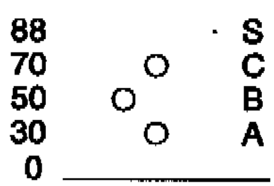
From the line constants program, we obtained the resistance and reactance data contained in matrix ZH, for the horizontal transmission line configuration, where the conductors are 40 feet above the earth, and spaced 20 feet apart. Two shield wires are centered 15 feet above the plane of the phase conductors.



$$ZH := \begin{bmatrix} 22.95 + w \cdot .3248j & 10.15 + w \cdot .1226j & 10.01 + w \cdot .1004j \\ 10.15 + w \cdot .1226j & 23.18 + w \cdot .3246j & 10.15 + w \cdot .1226j \\ 10.01 + w \cdot .1004j & 10.15 + w \cdot .1242j & 22.95 + w \cdot .3248j \end{bmatrix}$$

$$ZH = \begin{bmatrix} 22.95 + 122.45j & 10.15 + 46.22i & 10.01 + 37.851i \\ 10.15 + 46.22i & 23.18 + 122.374i & 10.15 + 46.22i \\ 10.01 + 37.851i & 10.15 + 46.22i & 22.95 + 122.45i \end{bmatrix}$$

From the line constants program, we similarly obtained the matrix ZV for the vertical configuration, with horizontal offsets of 0 for the shield, 16 for A, 20 for B and 16 for C. This is one side of a double circuit line.



$$ZV := \begin{bmatrix} 22.17 + w \cdot .3253j & 9.14 + w \cdot .1231j & 9.05 + w \cdot .1019j \\ 9.14 + w \cdot .1231j & 21.92 + w \cdot .3264j & 8.95 + w \cdot .1242j \\ 9.03 + w \cdot .1019j & 8.95 + w \cdot .1242j & 21.81 + w \cdot .3274j \end{bmatrix}$$

$$ZV = \begin{bmatrix} 22.17 + 122.638i & 9.14 + 46.409i & 9.05 + 38.416i \\ 9.14 + 46.409i & 21.92 + 123.053i & 8.95 + 46.823i \\ 9.03 + 38.416i & 8.95 + 46.823i & 21.81 + 123.43i \end{bmatrix}$$

Transform the phase impedance matrices into sequence impedance matrices, by pre-multiplying by matrix A and post-multiplying by AS.

$$ZSH := A \cdot ZH \cdot AS$$
$$ZSV := A \cdot ZV \cdot AS$$

$$ZSH = \begin{bmatrix} 43.233 + 209.285i & 2.333 - 1.489i & -2.456 - 1.276i \\ -2.456 - 1.276i & 12.923 + 78.994i & -4.846 + 2.817i \\ 2.333 - 1.489i & 4.862 + 2.788i & 12.923 + 78.994i \end{bmatrix}$$

$$ZSL = \begin{bmatrix} 40.053 + 210.806i & 2.342 - 1.729i & -2.055 - 1.614i \\ -2.045 - 1.608i & 12.923 + 79.163i & -4.715 + 2.766i \\ 2.352 - 1.735i & 4.732 + 2.714i & 12.923 + 79.152i \end{bmatrix}$$

In these matrices, the diagonal contains Z00, Z11 and Z22, and the off-diagonal terms are the mutual impedances between the sequence networks. Because the matrices did not diagonalize by the sequence transform, we know that the sequence networks are not independent.

Define a matrix function to find the average of the diagonal elements.

$$DIAG(MAT) := \tfrac{1}{3} \cdot tr(MAT)$$

Define a matrix function to find the average of the off-diagonal elements.

$$OFFDIAG(MAT) := 1/6 \cdot tr(MAT \cdot (1 - I))$$

Define a matrix function to calculate the magnitude of each element of MAT, normalized by scalar SF:

$$MAG(MAT, SF) := \left[\left| \frac{MAT}{SF} \right| \right]$$

Using these functions makes it easy to find the average self and mutual impedances from the phase impedance matrices, and to normalize any matrix for comparison purposes.

Beginning with the vertical configuration, find the average self impedance per phase, the average mutual impedance per phase, and find the average mutual impedance per phase normalized to the average self impedance.

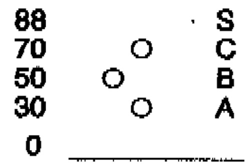
ZSSV: = DIAG(ZV)	ZSSV	= 21.967 + 123.04i	Average self impedance
ZMMV: = OFFDIAG(ZV)	ZMMV	= 9.043 + 43.833i	Average mutual impedance

MAG(ZMMV, ZSSV) = 0.358 — Magnitude of mutual impedance, normalized by self impedance.

This factor of 0.358 indicates that each of the currents in the other phases has about one-third of the effect on the voltage drop in the phase under consideration, as does the current in that phase itself.

Now normalize the matrix ZV by the average self impedance:

$$MAG(ZV,\ ZSSV) = \begin{bmatrix} 0.997 & 0.378 & 0.316 \\ 0.378 & 1 & 0.381 \\ 0.316 & 0.381 & 1.003 \end{bmatrix}$$



The self impedances of the three phases are nearly equal; however, the mutual impedances differ substantially from the average value found earlier (0.358).

Using the fact that Z11=Z22=ZSS-ZMM, normalize the sequence matrix by the positive sequence impedance and find the magnitude of each term in the matrix.

$$MAG(ZSV,\ ZSSV - ZMMV) = \begin{bmatrix} 2.675 & 0.036 & 0.033 \\ 0.032 & 1 & 0.068 \\ 0.036 & 0.068 & 1 \end{bmatrix}$$

We observe there are several percent of coupling between the sequence networks, owing to the assymetry of the conductor configuration.

Repeating the process for the horizontal configuration gives the results below:

ZSSH	:	DIAG(ZH)	ZSSH	=	23.027 + 122.424i
ZMMH	:	OFFDIAG(ZH)	ZMMH	=	10.103 + 43.43i
Z1	:	ZSSH - ZMMH	Z1	=	12.923 + 78.994i
Z0	:	ZSSH + 2 · ZMMH	Z0	=	43.233 + 209.285i

$MAG(ZMMH,\ ZSSH) = 0.358$

$$MAG(ZH,\ ZSSH) = \begin{bmatrix} 1 & 0.38 & 0.314 \\ 0.38 & 1 & 0.38 \\ 0.314 & 0.038 & 1 \end{bmatrix}$$

$$MAG(ZSH,\ ZSSH - ZMMH) = \begin{bmatrix} 2.67 & 0.035 & 0.035 \\ 0.035 & 1 & 0.07 \\ 0.035 & 0.07 & 1 \end{bmatrix}$$

Similar off-diagonal terms exist, as compared to the vertical configuration. One difference is that the phase impedance matrix self impedance terms are nearly the same, as the only assymetry of these conductors with respect to ground is the closer relationship of the center phase (B) to the two ground wires. The difference shows up only in the fourth decimal place.

To further analyze the effects of line unbalance, form a matrix ZHBAL to represent a balanced line, which a fault locator set with Z0 and Z1 would assume exists.

$$ZHBAL := \begin{bmatrix} ZSSH & ZMMH & ZMMH \\ ZMMH & ZSSH & ZMMH \\ ZMMH & ZMMH & ZSSH \end{bmatrix}$$

To see the difference between the actual line and the balanced approximation, take the difference between matrices ZHBAL and ZH, and normalize the matrix elements by the magnitude of the nominal self impedance.

$$DZH := \left[\frac{1}{|ZSSH|} \right] \cdot (ZHBAL - ZH)$$ ... balanced vs unbalanced representations

$$DZH := \begin{bmatrix} 0.001 & -0.022i & 0.001 \; +0.045i \\ -0.0221i & -0.001 & -0.022i \\ 0.001 \; +0.045i & -0.022i & 0.001 \end{bmatrix}$$

The very small diagonal elements reconfirm our earlier observation that the self impedances are very close together. The off-diagonal elements are two to four percent, meaning that one unit of current in one phase indices a voltage difference of two to four percent between the actual line and its balanced model, and that voltage is induced in a different phase.

Consider an AG fault, with no load flow, on a radial system driven by an infinite bus. IB=IC=0, and IA=VA/ZSS=1.

$IA := e^{-j \cdot 79.3 \deg}$ $\quad IB := 0 \quad IC := 0$

$$\begin{bmatrix} DVA \\ DVB \\ DVC \end{bmatrix} := DZH \cdot \begin{bmatrix} IA \\ IB \\ IC \end{bmatrix} \qquad DVA = -8.398 \cdot 10^{-5} - 0.001i$$

The very small difference between the voltages calculated with the unbalanced and balanced models indicates that virtually no error will result from using the balanced model instead of the unbalanced model in this special case.

Next, we try the case of just balanced load, to determine the voltage difference between the unbalanced and balanced representations.

Balanced load:

$$\begin{bmatrix} DVA \\ DVB \\ DVC \end{bmatrix} := DZH \cdot \begin{bmatrix} 1 \\ a^2 \\ a \end{bmatrix} \qquad \begin{aligned} DVA &= -0.058 - 0.01i \\ DVB &= 0.02 - 0.011i \\ DVC &= -0.019 + 0.057i \end{aligned}$$

The balanced model produces a six percent less voltage drop in the outside phase (A and C) than the unbalanced model. The sources of these differences are the large differences in the "a-c" mutual elements.

The center phase is not as affected by phase a and phase c currents, as we might expect, since a and c are symmetrical with respect to b.

This balanced load analysis is important, because it shows that load currents flowing in the uninvolved phases induces voltages in the faulted phase, which are not accounted for by the balanced model. When load flows outward, a fault on either of the outside phases tends to look closer in, and a fault on the inner phase tends to look farther away.

Repeating the calculations for the vertical configuration yield the results below:

$$ZVBAL := \begin{bmatrix} ZSSV & ZMMV & ZMMV \\ ZMMV & ZSSV & ZMMV \\ ZMMV & ZMMV & ZSSV \end{bmatrix}$$

$$DZV := \left[\frac{1}{|ZSSV|} \right] \cdot (ZVBAL - ZV) \quad \text{... balanced vs unbalanced representations}$$

$$\begin{bmatrix} DVA \\ DVB \\ DVC \end{bmatrix} := DZV \cdot \begin{bmatrix} IA \\ IB \\ IC \end{bmatrix} \qquad DVA = 0.003 + 0.002i$$

The 0.003 pu larger voltage for the balanced case maps into a 0.3% increase in the indicated distance to fault, and is due to the very slightly lower self impedance of the lowest conductor, as compared to the average. If we were to fault phase B, the error would be almost zero, and the phase c error is about 0.3% short.

The load case is very similar to the horizontal configuration, and the same comments hold.

$$\begin{bmatrix} DVA \\ DVB \\ DVC \end{bmatrix} := DZV \cdot \begin{bmatrix} 1 \\ a^2 \\ a \end{bmatrix} \qquad \begin{aligned} DVA &= -0.057 - 0.008i \\ DVB &= 0.019 - 0.008i \\ DVC &= -0.019 + 0.057i \end{aligned}$$

Phase-to-phase faults are measured by the loop impedances, which average out to the positive-sequence impedances. However, there is substantial difference between the loop impedances A-B and A-C, for example. For the horizontal line configuration, consider the impedance ZAA-ZAB, which should be used to locate an AB fault, and compare it to the positive-sequence impedance. Do the same for the ac loop.

$$ZAA := ZH_{0,0} \qquad ZAB := ZH_{0,1} \qquad ZAC := ZH_{0,2}$$

$$\frac{ZAA - ZAB}{ZSSH - ZMMH} = 0.966 - 0.004i$$

$$\frac{ZAA - ZAC}{ZSSH - ZMMH} = 1.069 + 0.011i$$

This shows that the correct loop impedance for an AB fault is about three percent less that the positive-sequence impedance, so that a fault locator scaling on the basis of the positive-sequence impedance will indicate short by several percent.

On the other hand, a fault between the outside phases will indicate long, since the actual value of impedance is greater than the positive-sequence impedance.

Conclusions

1. The horizontal line configuration offers little ground fault error, when the unfaulted phase currents are small.

2. The vertical line configuration offers a small ground fault error even when the currents are zero in the uninvolved phases, since the conductors have different self impedances.

3. Load currents or faults currents in the uninvolved phases can induce voltages several percent different than predicted by the balanced model. These differences can cause significant fault-locating errors.

4. Phase fault locating accuracy depends significantly on the involved loop. Errors of several percent are possible, due to the substantial differences in the loop impedances.