Philosophies for Testing Protective Relays

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PHILOSOPHIES FOR TESTING PROTECTIVE RELAYS

INTRODUCTION

Utility engineers budget resources for testing and maintenance of protective relays and distribute the available testing personnel and equipment accordingly. To efficiently allocate test and maintenance resources, utility engineers must consider the unique requirements of various types of protective relays.

This paper evaluates traditional testing philosophies to determine their effectiveness when applied to new relay designs. The paper statistically illustrates the differences in optimum test intervals between traditional relay designs and new relay designs. Periodic testing adds very little availability to relays with self-testing.

TRADITIONAL AND DIGITAL RELAYS

For this discussion, we will refer to two different types of protective relays. Relays which include self-testing, alarms, and event reporting we refer to as digital relays. Those which do not include these features are referred to as traditional relays.

WHY TEST PROTECTIVE RELAYS?

The goal of protective relay testing is to maximize the availability of protection and minimize risk of relay misoperation. With this in mind, we must define adequate testing methods and intervals for the various types of protective relaying equipment.

When a traditional relay fails, the failure can cause the relay to false trip, prevent operation for a fault, or alter the relay operating characteristics. Traditional relays do not provide self-tests or status monitoring; therefore, routine testing is required to verify proper operation. If a problem exists in a traditional relay, the problem may go undetected until routine maintenance is performed or the relay fails to operate for a fault. The reliability of the traditional relay is, therefore, largely dependent on the frequency of routine maintenance.

Digital relay failures can also cause relay misoperations and prevent operation for faults. However, relay characteristics are typically not affected by failures. Failures tend to be significant enough to either generate a self-test failure indication or cause the user to recognize the problem during normal use of the relay.

DIGITAL RELAY SELF-TESTS

As a minimum, digital relay self-tests include tests of memory chips, a/d converter, power supply, and storage of relay settings. These periodic self-tests monitor the status of the digital relay and close an alarm contact when a failure is detected. Additionally, the digital relay may disable trip and control functions upon detection of certain self-test failures. Since the relay self-tests are executed often in the digital relay, component failures are usually discovered when the failure occurs.

TYPE TESTING

When a utility engineer selects a new relay design, it is essential to perform tests of the selected relay to ensure proper operation for the intended application. These tests are referred to as type tests and are usually implemented on a single representative relay from the manufacturer. During type tests, utility staff are introduced to new relay models and functions. If there are specific application questions, utility staff discuss these questions with the relay manufacturer until there is a clear understanding of all the protective functions. Type tests include detailed tests of the relay characteristics such as mho circle plots, time-overcurrent curve plots, relay element accuracy, etc. The main objective of the type test is verification of the relay algorithms and characteristics.

COMMISSIONING TESTING

Utilities typically require tests of each relay prior to placing relays in service. These tests are referred to as commissioning or installation tests.

Once the utility accepts the results of the digital relay type tests, the requirement for commissioning testing is reduced. The operating characteristics of microprocessor-based relays are consistent. This allows us to rely on the type tests for detailed characteristic tests and focus the commissioning tests on simple tests of the relay hardware.

Digital relay commissioning tests may be limited to include tests for calibration, input/output functionality, simple element accuracy tests, etc. Commissioning tests should also verify the effectiveness of calculated relay element and logic settings. Greater reliance on the type tests for the detailed relay characteristic tests is well justified because those characteristics are fixed in the relay algorithms.

ROUTINE MAINTENANCE TESTING

Routine testing of protective relays has been the primary method of detecting failures in traditional relays. The only other way of determining that a traditional relay has failed is to observe a misoperation. Routine testing is scheduled based on utility experience with the devices in question. However, there is risk involved, both with performance of the test and with leaving the relay untested.

The goal of routine maintenance is to verify that the protective relay will not operate unnecessarily and will operate when required.

Typically, routine maintenance is performed periodically with a specified interval between tests. A common belief is that a shorter test interval increases overall system reliability; there are limitations to this statement, however. There is the possibility that a system failure could be introduced while performing routine maintenance. Performing a routine test creates the risk that a functioning relay might be damaged by the tests or might be left in an unserviceable condition following the test.

The time between tests is typically measured in years. If a failed relay does not misoperate in that period, its failure goes unnoticed and unrepaired for what may be a significant portion of the testing interval. So, the risk of leaving the relay untested is that it may not properly operate when necessary.

To schedule routine testing, the utility engineer must balance risks: the risk of leaving a failed relay in service versus the smaller risk of damaging a sound relay.

How can routine testing find problems in protective relays? In order to find problems that might be present, it is helpful to examine the type of problems that can occur in both classes of relays. Then, examine the types of tests being performed to see if they are exercising the relays in meaningful ways.

Routine Testing Traditional Relays

Traditional relays are often built with induction disks or cylinders that turn on jewel bearings. Heavy-duty resistor, inductor, and capacitor networks shape operating characteristics. Springs and levers define operating times. Tests of traditional relays necessarily check the operating characteristics that are affected by the individual components: pickup settings, operating times, and characteristics.

If routine testing detects a problem with a traditional relay, we can only find the last time the relay was shown to operate properly in a fault record or a test report. The relay could have failed on the day following the last correct operation, on the day before this misoperation, or on any day in between.

Routine Testing Digital Relays

Digital relays are built using a microprocessor, an ac signal data acquisition system, memory components containing the relay algorithms, contact inputs to control the relay, and contact outputs to control other equipment. Digital relay operating characteristics are defined by the algorithms and settings contained in the relay memory.

Digital relays are often equipped with automatic self-test functions. These self-tests verify correct operation of critical relay components. If a self-test detects an abnormal condition, it can close an output contact, send a message, or provide some other indication of the failure. When the alarm occurs, a technician can be dispatched to repair or replace the device quickly.

It is helpful to define the requirements of digital relay routine maintenance by dividing the hardware into three categories and specifying maintenance practices which adequately test each section. For the purposes of testing, it is convenient to divide the relay into the following three sections:

1) Analog Input Section
2) Processing Section
3) Contact Input/Output Circuitry

The analog input section is typically monitored by automatic self-testing. This may be somewhat limited because a steady-state condition cannot be fully defined. With a protective relay, there are often many steady-state conditions possible under each mode of operation. Since the analog input portion of the digital relay is only partially self-tested, routine maintenance assists in verification of the analog measuring components.

Many digital relays offer metering features which give the user a convenient means of verifying the accuracy of the relay analog input section. The user can verify metering quantities and be assured the relay is using valid data for its relay element computations. This practice is sound if the digital relay uses the same measuring circuitry for both metering and relaying. On the other hand, if the relay uses separate circuitry for its metering functions, the metering data checks only the components common to both the metering and relaying circuitry.

The contact input/output circuitry is another part of the digital relay which allows only partial automatic testing. For this reason, it may be appropriate to implement a routine trip check. Many digital relays provide a trip feature which allows the user to locally or remotely trip the relay. The trip check verifies the trip circuit wiring and the integrity of the trip coil. This trip command feature provides a convenient means of tripping the circuit breaker without the need to inject a simulated fault to the relay. If the relay is routinely operating for faults, the actual relay operations may be adequate verification of the relay input/output functions.

The digital processing section, typically a microprocessor, is the interface between the analog input section and the contact input/output section. Since the analog and contact input/output sections cannot function without the processing section, normal relay use and maintenance checks act as routine verification of the microprocessor. Additionally, manufacturers are able to offer very thorough self-tests to continually monitor the status of the computer.

Utility engineers should work closely with relay vendors to determine what relay functions are not checked by relay self-tests and how those functions should be checked in the field. In the case of the processing section, there are typically no special tests required.

Many of the maintenance features are executable by remote command and often could replace routine maintenance altogether. Also, consider the analysis of digital relay fault data comparable to routine relay maintenance. Those relays which do not encounter faults may require more thorough routine maintenance checks.

Because the digital relay provides an indication when a problem occurs, the possibility that a failed digital relay could remain in service for a significant amount of time is reduced. If the utility monitors relay self-test alarm contacts, a failed relay can generally be repaired or replaced within hours or days of a failure.

Digital Relay Data Analysis

Used properly, event reporting features provided by digital relays supplement routine maintenance. Event reports typically provide a record of each relay operation with the same resolution as the sample rate of the digital relay. If testing personnel devote a small percentage of their time to analyzing these fault records, they can find relay problems displayed in the event report data. The analysis of actual fault data is a true test of the instrument rather than a simulated test. Careful analysis of relay event reports indicates problems which could otherwise go undetected due to limitations of digital relay self-tests.

Event reports can also indicate problems external to the digital relay. Transformers, trip circuits, communication equipment, auxiliary input/output devices are examples of external equipment which may be indirectly monitored using the event report.

Some simple guidelines assist in detecting problems using event reports:

1) Does the indicated fault location agree with actual fault location?

If the relay indicates a fault location that does not agree with the actual fault location, the user is alerted of possible impedance setting errors, instrument transformer ratio problems, etc.

2) Was the fault in the expected zone of operation?

Further analysis of the fault location helps check the relay reach settings. The fault location should match the expected zone of operation. If the fault location and zone of operation do not agree, fault resistance may have influenced the relay element operation for the fault.

3) Did the expected relay elements operate?

The event report shows each analog channel and the status of each relay element for the full duration of the report. If an element is improperly set or operates incorrectly, the error is immediately apparent in the event report data.

4) Is the breaker operating time reasonable?

The breaker operating time may be monitored by comparing the trip output to the breaker status input. Slow or inconsistent breaker operating times may indicate a breaker in need of maintenance.

5) Are prefault voltage and current measurements reasonable?

Event reports typically record a small portion of prefault data. The prefault information allows the user to analyze the integrity of the system before the fault occurred.

6) Does load flow agree with system data?

Load flow from the prefault data may be compared to other fault recording instruments to verify that the direction of load flow agrees in all fault recording instruments. If load flow does not agree, there is most likely a connection error in one of the instruments.

SELECTING THE OPTIMUM TEST INTERVAL

Several IEEE papers [1,2] describe probabilistic methods of determining the optimum test interval for traditional relays. Anderson and Agarwal [1] propose a calculation method that produces several probability measures. Two measurements of interest are Abnormal Unavailability and Protection Unavailability. The Abnormal Unavailability reflects the result of a fault occurring while the protective relay is out of service. The Protection Unavailability is the probability that the relay will be out of service. The relay could be out of service because of a failure, testing, or repairs.

The model shown in [1] makes the following assumptions regarding the relays modelled:

1) An inspection or fault must occur in order to detect a relay failure.

2) A relay must be taken out of service to be inspected.

3) The time required to test a relay is equal to the time required to repair or replace a failed relay.

4) Inspection of the protection always detects failures and does not cause failures.

5) Repair always restores the protection to good as new.

Assumptions 1 and 2 make the model primarily applicable to traditional relays, those relays without self-tests. Assumption 3 simplifies the model calculations without detracting appreciably from the results.

Appendix A presents a new model based on [1] that accounts for the beneficial effects of relay self-tests. The new model has one additional state which represents relay failures detected by self-tests. These failures can be discovered immediately and repaired without waiting for the next fault or inspection.

From the model, we can calculate the Abnormal Unavailability and Protection Unavailability of relays with or without self-tests by adjusting the transition rates that define the model. The transition rates are defined in Table 1.

Table 1: Reliability Model Transition Rates	
F_p	Relay failure rate, failures per year (reciprocal of MTBF)
R_r	Relay repair rate, repairs per hour
R_t	Relay test rate, tests per hour
F_c	Protected component failure rate, faults per year
R_c	Protected component repair rate, repairs per hour
F_{cc}	Common-cause failure rate of the relay and protected component
S_n	Normal switching rate, operations per hour
S_b	Backup switching rate, operations per hour
S_m	Manual switching rate, operations per hour
Θ_{pm}	Relay inspection rate, inspections per hour (reciprocal of test interval)

Unless otherwise noted, the model uses the following transition rates:

R_r = 0.5 relay repairs per hour
R_t = 1.0 relay routine test per hour
R_c = 0.5 component repairs per hour
F_{cc} = 1.0 common-cause failure per million hours
S_n = 43200 operations per hour (corresponds to 5 cycle switching speed)
S_b = 21600 operations per hour (corresponds to 10 cycle backup switching speed)
S_m = 0.5 operations per hour (2 hours to isolate component after backup operation)

Figure 1 shows the Abnormal Unavailability versus routine test interval for a system using a traditional relay that does not have self-testing. The plot is for a relay with a Mean Time Between Failures (MTBF) of 50 years monitoring a line that is faulted twice per year. The optimum routine test interval is the point where Abnormal Unavailability is lowest: approximately 700 hours or one month. When the test interval is shorter, the relay is often out of service due to testing. In this area, the relay is being tested too much and is likely to miss any fault that occurs. When the test interval is longer, the relay becomes more likely to be out of service because of an undetected problem with the relay: the relay is being tested too little.

The model results indicate, to achieve the highest reliability, the relay test interval should be much shorter than the interval between faults. They also suggest that, if possible, the relay should be left in service while the tests are performed. This is precisely what automatic self-tests do for digital relays: test often and test without disturbing the protection.

Figures 2 and 3 show the sensitivity of the Abnormal Unavailability for a traditional relay to the number of faults per year and to the relay MTBF, respectively. Figure 2 shows traces for systems responding to one and ten faults per year. We see that the number of faults per year has the largest effect on the Abnormal Unavailability when the test interval is extremely low. The optimum test interval is not appreciably influenced by the number of faults per year.

Figure 3 shows that MTBF has the greatest effect on Abnormal Unavailability when the routine test interval is long. This is reasonable. With a low MTBF and a long test interval, the relay is more likely to have experienced an undetected failure when a fault occurs.

Figure 4 compares relays with and without self-tests on the basis of Protection Unavailability. Figure 4 shows traces representing four types of relay self-tests. When ST = 0%, the relay is not equipped with self-testing. When ST = 50%, the relay self-tests detect half of all relays failures immediately. When ST = 90% and 99%, the relay self-tests detect 90% and 99% of relay failures, respectively. Failures not detected by self-tests are only detected when the relay is tested or when a misoperation occurs.

Figure 4 shows that a traditional relay (ST = 0%) is ten times as likely as a digital relay with 90% self-tests to be out of service due to a relay failure when the routine test interval is 10^5 hours (approximately 11 years). The traditional relay is 100 times as likely as the relay with 99% self-tests to be out of service. In addition, the relay featuring 99% self-tests shows a decreasing Protection Unavailability as the test interval increases. This relay is less likely to miss a fault if the test interval is longer. This yields a surprising result: to improve availability, test such a relay less frequently. Figure 5 shows the Abnormal Unavailability of the four systems.

Figure 6 shows that, for a relay with self-tests, Abnormal Unavailability is not appreciably affected by the frequency of faults, for long routine maintenance intervals. The plot shows performance for one and ten faults per year.

Figure 7 shows another surprising result. The plot shows Abnormal Unavailability for two systems using relays with self-tests. One relay has an MTBF of 10 years, the other has an MTBF of 100 years. The plot shows that the system with the low MTBF relay has only a slightly higher Abnormal Unavailability. The benefit of a long MTBF is that reliable relays do not need to be repaired or replaced as often as relays with a short MTBF. This saves maintenance time and money. Thus, a long MTBF is valuable in saving money on repairs, but is not very important to availability.

Figure 8 compares Abnormal Unavailability for a power system protected by traditional relays to a power system protected by digital relays. For this plot, the two digital relays have MTBF of 10 and 100 years, self-test effectiveness of 95%, and a test time of four hours. The traditional relay has no self-tests, an MTBF of 50 years, and a test time of eight hours. This chart shows that a traditional relay terminal tested once every four months is not as reliable as a digital relay terminal tested every 40 years.

CONCLUSION

The features of digital relays reduce routine tests to a very short list: meter checks and input/output tests. Routine characteristic and timing checks are not necessary for digital relays. Probability analysis shows that relays with self-tests do not need to be routine tested like relays without self-tests. If the relay is measuring properly, and no self-test has failed, there is no reason to test the relay further.

Use the digital relay reporting functions as maintenance tools. Event report analysis should supplement or replace routine maintenance checks of relays with self-tests. Event report analysis increases a tester's understanding of the digital relay and of the power system.

Because self-tests quickly indicate the vast majority of relay failures, the MTBF of a digital relay does not have a large impact on the power system Abnormal Unavailability. When a relay is equipped with self-tests, the benefit of a high MTBF is that fewer relays need replacement or repair. A high MTBF saves maintenance time and money. Relay self-testing saves routine testing time.

When a relay is not equipped with self-tests, a high MTBF and a short test interval are both essential to minimize system Abnormal Unavailability.

Reducing the complexity and frequency of digital relay routine tests saves labor. The labor resources can be applied to more frequent and complete tests of traditional relays. The result will be higher overall reliability and availability from all relays, both digital and traditional.

REFERENCES

1. "An Improved Model for Protective-System Reliability," by P. M. Anderson and S. K. Agarwal, Power Math Associates, Inc., IEEE Transactions on Reliability Volume 41, No. 3, 1992 September.

2. "Graph Theory Concepts in Frequency and Availability Analysis," by Sudhir Kumar and Roy Billinton, University of Saskatchewan, IEEE Transactions on Reliability Volume R-34, No. 4, 1985 October.

FIGURE 1 - Selecting the Optimum Test Interval

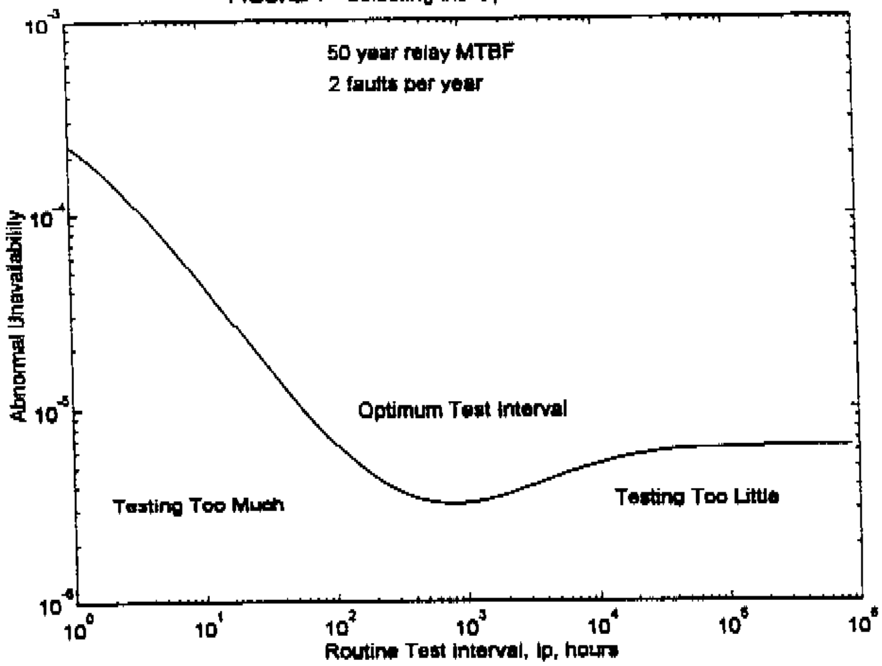


FIGURE 2 - Sensitivity to Faults/Year, Relay Without Self-Tests

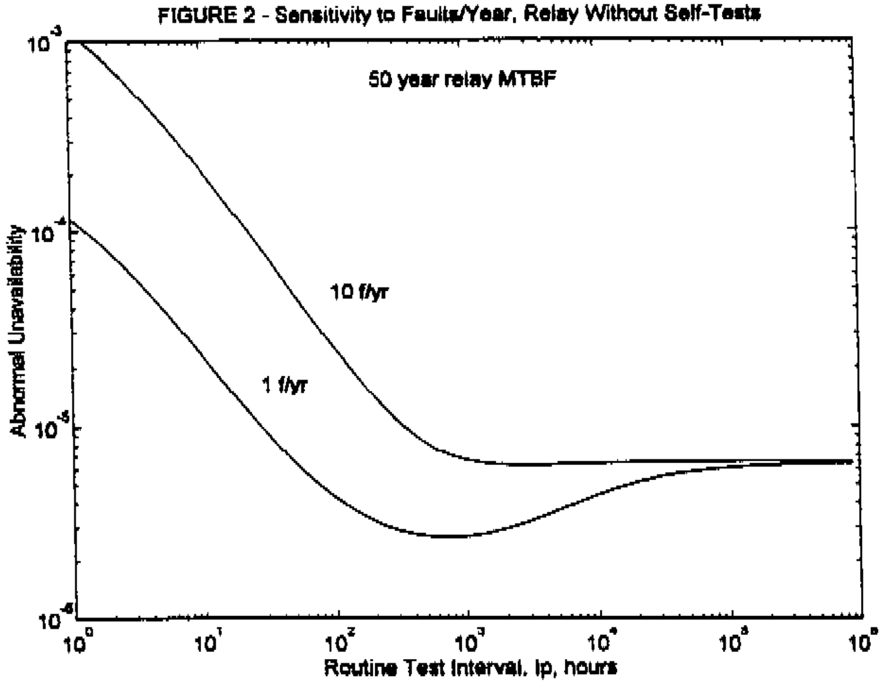


FIGURE 3 - Sensitivity to MTBF, Relay Without Self-Tests

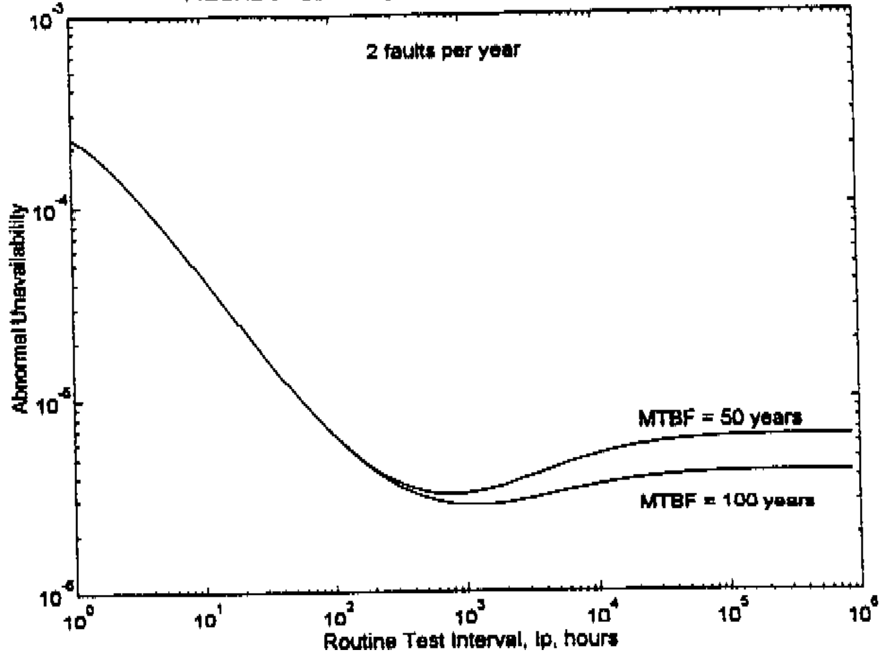


FIGURE 4 - Effect of Self-Tests on Protection Unavailability

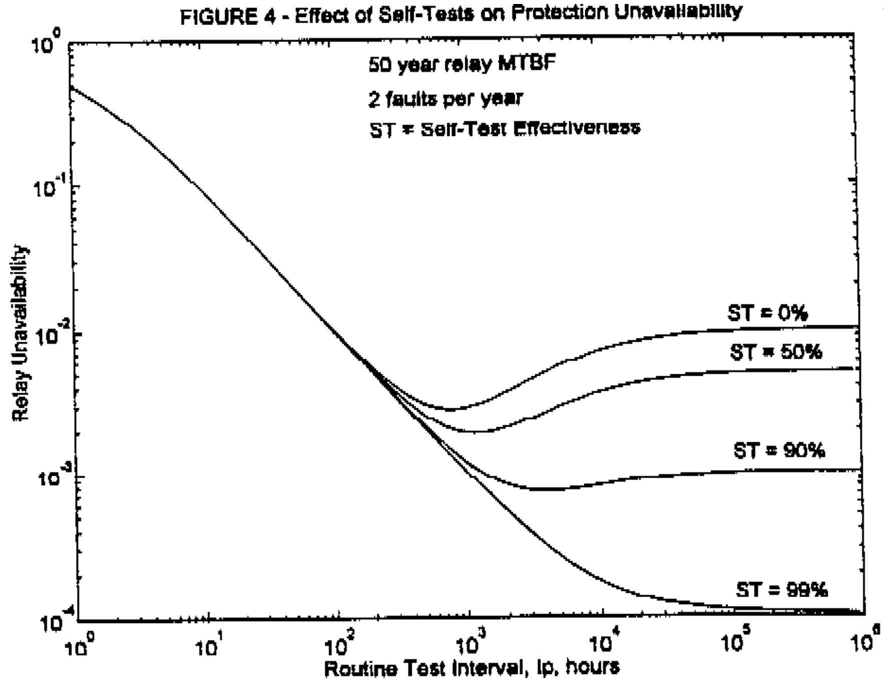


FIGURE 5 - Effect of Relay Self-Tests on Power System Unavailability

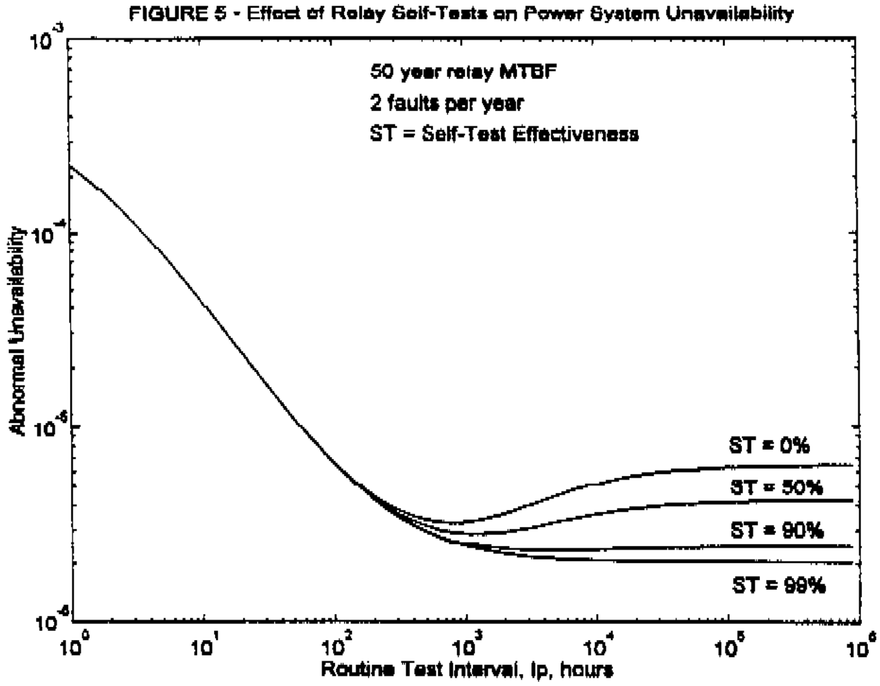


FIGURE 6 - Sensitivity to Faults per Year, Relay With Self-Tests

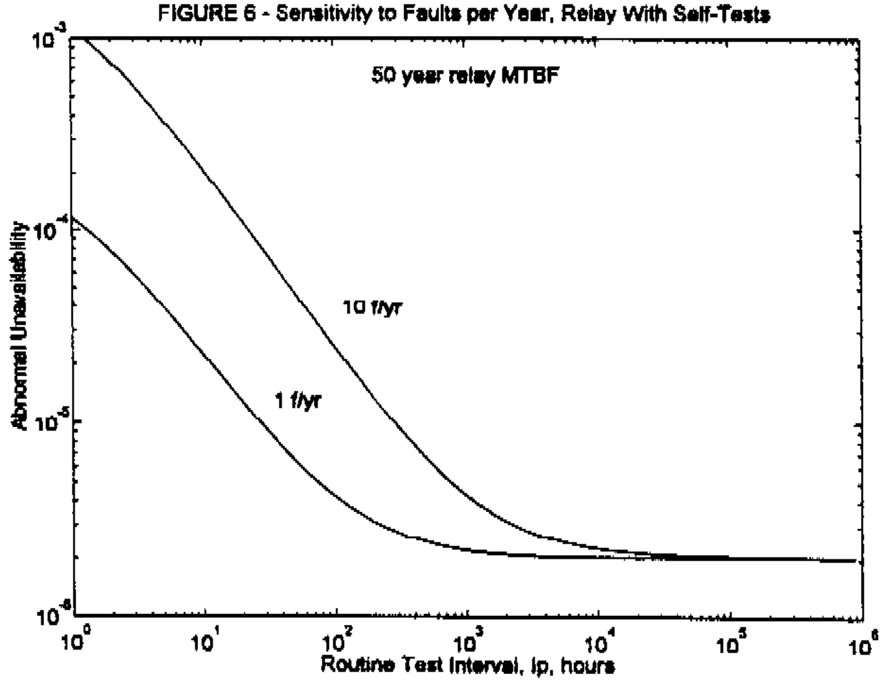


FIGURE 7 - Sensitivity to MTBF, Relay With Self-Tests

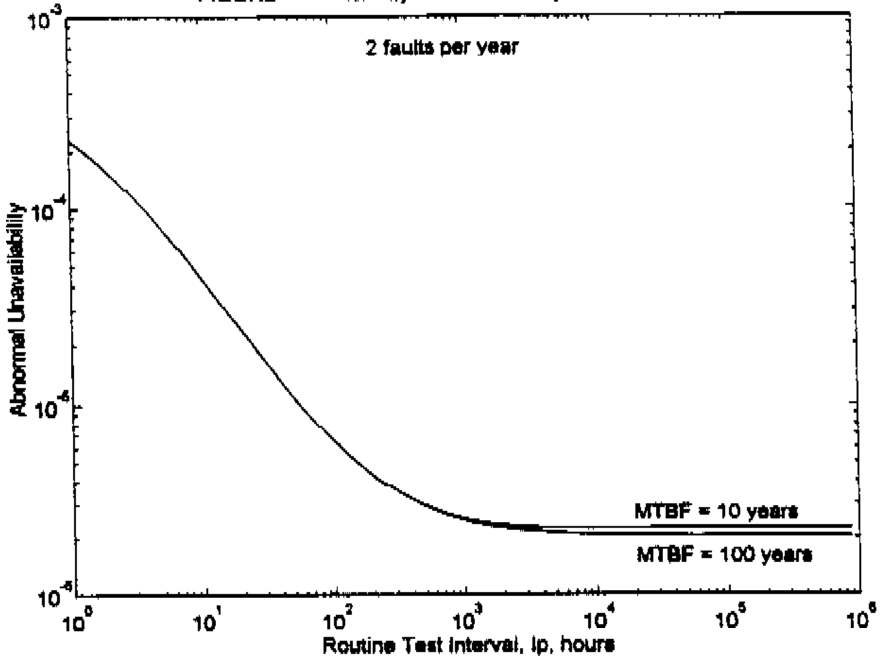
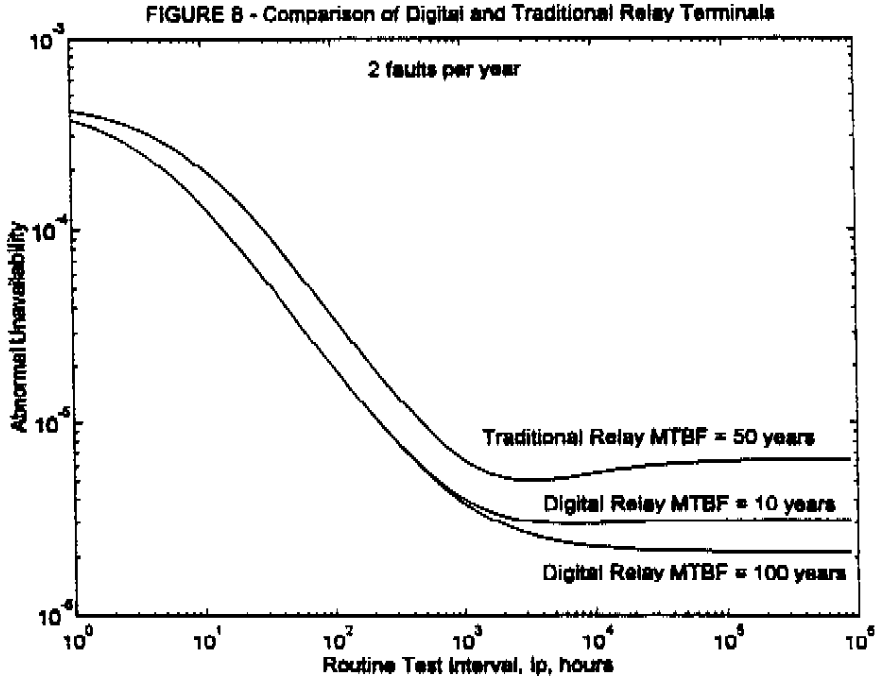


FIGURE 8 - Comparison of Digital and Traditional Relay Terminals



BIOGRAPHIES

Edmund O. Schweitzer, III is President of Schweitzer Engineering Laboratories, Inc., Pullman, Washington, U.S.A., a company that designs and manufacturers microprocessor-based protective relays for electric power systems. He is also an Adjunct Professor at Washington State University. He received his BSEE at Purdue University in 1968 and MSEE at Purdue University in 1971. He earned his PhD at Washington State University in 1977. He has authored or co-authored over 30 technical papers. He is a member of Eta Kappa Nu and Tau Beta Pi.

Daqing Hou received BS and MS degrees in Electrical Engineering from the Northeast University of Technology, China, 1981 and 1984, respectively. He received his PhD in Electrical and Computer Engineering from Washington State University in 1991. Since 1990, he has been with Schweitzer Engineering Laboraties, Inc., Pullman, Washington, U.S.A., where he is currently an application engineer. His work includes system modeling, simulation and signal processing for power system digital protective relays. His research interests include multivariable linear systems, system identification, and signal processing. Hou is a member of the IEEE and has authored or co-authored several technical papers.

John J. Kumm received his BSEE degree at the University of Idaho in 1989. Since 1989, he has been with Schweitzer Engineering Laboratories, Inc., Pullman, Washington, U.S.A., where he is currently an application engineer. His work includes new product specification and product support. He is a member of IEEE Power Engineering Society.

Mark S. Weber received his AAS degree in Electronics Engineering Technology in 1985. Since 1986, he has been with Schweitzer Engineering Laboratories, Inc., Pullman, Washington, U.S.A. His work includes product support, reliability analysis, and testing of digital protective relays manufactured at SEL.

APPENDIX A

Reference [1] proposes an eight-state model defined by the operating condition of the relay and the protected component. The eight-state model does not account for relay self-testing. Figure A.1 shows a nine-state model that accounts for self-testing. The model is divided into four quadrants representing the condition of the relay (Protection) and the line (Component).

State 1 represents the normal operating condition where the line is energized (Component UP) and the relay is operating properly (Protection UP). When a line fault occurs, the Component makes the transition to a down state represented by State 2. In State 2, the line is faulted, but the relay is operating properly and signals the circuit breaker to trip. The normal switching transition takes the model system to State 6 where the line is isolated. The line is then repaired and re-energized, taking the model system back to State 1.

States 5, 3, and 9 represent conditions where the relay is out of service and unavailable to trip should a fault occur. In State 5, the relay is out of service being inspected. In States 3 and 9 the relay is out of service due to a relay failure. State 9 represents the relay under repair. The model system enters State 9 from State 1 when a relay failure is detected by the relay self-test function. The model system enters State 9 from State 3 when a relay failure is detected by a routine maintenance test. The model system enters State 3 from State 1 when a relay failure occurs that is not detected by the relay self-test function.

The effectiveness of self-testing can be varied in the model. The overall relay failure rate (Fp), is multiplied by a per unit factor (ST), to indicate the portion of all relay failures that are detected by self-test operation. The remainder of failures can only be detected by routine testing or by observing a misoperation. Digital relays with varying degrees of self-test effectiveness can be represented in the model by adjusting the value of ST.

The model system enters State 4 if a fault occurs while the relay is out of service, or if a common-cause failure of the relay and system occurs. If a fault occurs while the relay is out of service, the model assumes that remote backup protection must operate to isolate the fault. When the remote protection operates, a larger portion of the power system is taken out of service than would have been removed had the failed relay operated properly. This is represented in State 4 and State 8 by the isolation of C and X, where X is the additional equipment that was removed from service by the backup operation.

Calculate the probability that the system will reside in a given state using a Markov Transition Matrix or using the flow graph method [2]. We used a PC-based matrix calculation software, MatLab™, to perform the matrix calculations. All the transition rates must first be converted to operations per hour. The Markov Transition Matrix is assembled from the transition rates and manipulated as shown in the equations below. The resulting vector includes the probability of the system residing in any of the nine states.

The Abnormal Unavailability is the sum of probabilities P_4 and P_8. The Protection Unavailability is the sum of probabilities P_3, P_5, and P_9.

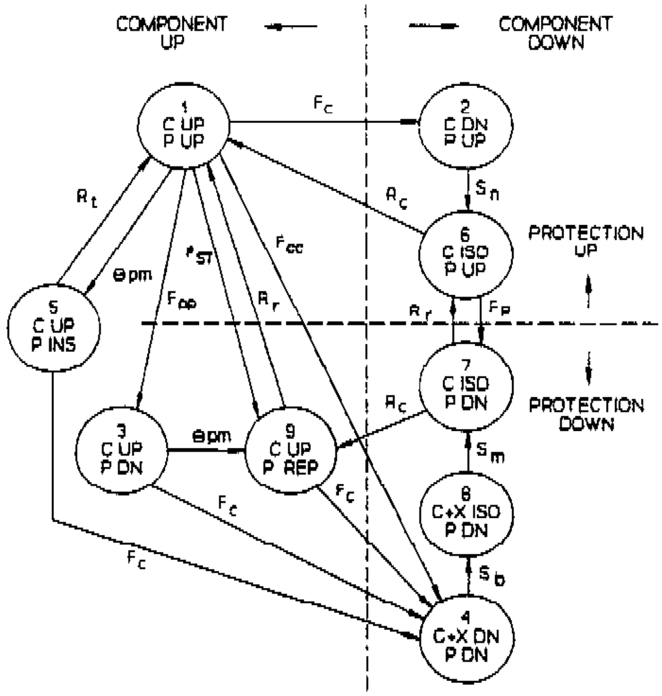


Figure A.1: Markov Model of a Protection/Component System, Relay Includes Self-Tests

__Markov Model Transition Rate Definitions__

Failure Rates:

- F_p Relay Failures (reciprocal of relay Mean Time Between Failures, MTBF)
- ST Self-test Effectiveness Index (per unit)
- F_{st} Relay Failures detected by self-test (F_p·ST), failures per year
- F_{pp} Relay Failures not detected by self-test (F_p·[1-ST]), failures per year
- F_c Component Failures, faults per year
- F_{cc} Common-cause failures of the relay and component, failures per year

Repair Rates:

- R_c Protected Component repairs per hour
- R_t Relay inspections per hour
- R_r Relay repairs per hour

Switching Rates:

- S_n Normal tripping operations per hour (reciprocal of normal fault clearing time)
- S_b Backup tripping operations per hour (reciprocal of backup fault clearing time)
- S_m Manual isolation operations per hour

Inspection Rate:

- I_{pm} Protection Inspection interval
- Θ_{pm} Protection Inspection rate ($1/I_{pm}$)

Unless otherwise noted, the model uses the following transition rates:

R_r = 0.5 relay repairs per hour
R_t = 1.0 relay routine test per hour
R_c = 0.5 component repairs per hour
F_{cc} = 1.0 common-cause failure per million hours
S_n = 43200 operations per hour (reciprocal of 5 cycle fault clearing time)
S_b = 21600 operations per hour (reciprocal of 10 cycle fault clearing time)
S_m = 0.5 operations per hour (2 hours to isolate component after backup operation)

Markov Transition Matrix for the nine state system shown in Figure A.1 is:

$$T = \begin{bmatrix} a_{11} & F_c & F_{pp} & F_{cc} & \theta_{pm} & 0 & 0 & 0 & F_{st} \\ 0 & a_{22} & 0 & 0 & 0 & S_n & 0 & 0 & 0 \\ 0 & 0 & a_{33} & F_c & 0 & 0 & 0 & 0 & \theta_{pm} \\ 0 & 0 & 0 & a_{44} & 0 & 0 & 0 & S_b & 0 \\ R_t & 0 & 0 & F_c & a_{55} & 0 & 0 & 0 & 0 \\ R_c & 0 & 0 & 0 & 0 & a_{66} & F_p & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & R_r & a_{77} & 0 & R_c \\ 0 & 0 & 0 & 0 & 0 & 0 & S_m & a_{88} & 0 \\ R_r & 0 & 0 & F_c & 0 & 0 & 0 & 0 & a_{99} \end{bmatrix}$$

$$a_{11} = 1 - (F_c + F_{cc} + F_{st} + F_{pp} + \theta_{pm})$$
$$a_{22} = 1 - S_n$$
$$a_{33} = 1 - (\theta_{pm} + F_c)$$
$$a_{44} = 1 - S_b$$
$$a_{55} = 1 - (R_t + F_c)$$
$$a_{66} = 1 - (R_c + F_p)$$
$$a_{77} = 1 - (R_r + R_c)$$
$$a_{88} = 1 - S_m$$
$$a_{99} = 1 - (R_r + F_c)$$
$$P^T = [P_1\ P_2\ P_3\ P_4\ P_5\ P_6\ P_7\ P_8\ P_9]$$
$$P^T \cdot T = P^T \quad \text{or} \quad P^T \cdot [T - I] = 0$$

where I = Identity Matrix

and $\sum_i P_i = 1$

Abnormal Unavailability, AbUn = $P_4 + P_8$

Protection Unavailability, ProtUn = $P_3 + P_5 + P_9$