

# Introduction to Symmetrical Components

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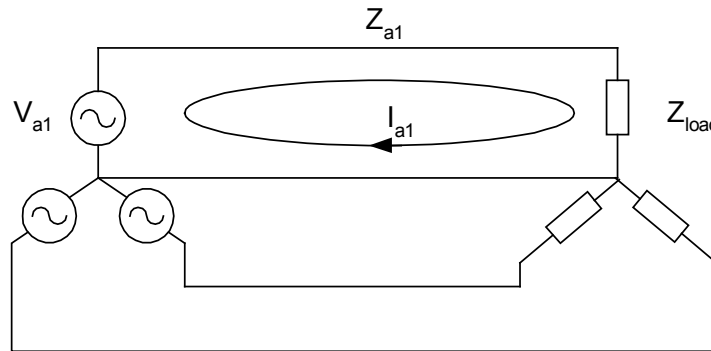
# INTRODUCTION TO SYMMETRICAL COMPONENTS

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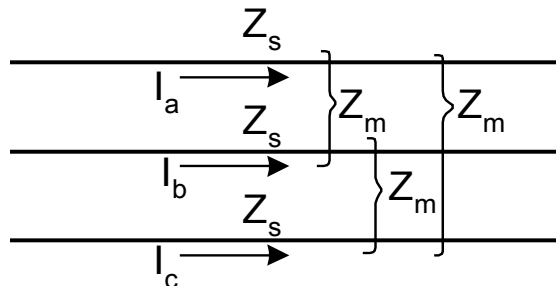
## SYMMETRICAL COMPONENTS

Figure 1 is the diagram of a balanced three-phase circuit. The impedance of the lines and loads are the same in each phase, and the source voltages are equal in magnitude and are 120 degrees apart. The balanced condition allows us to treat one phase as an independent single-phase circuit. Once we have calculated the current in the A-phase, the balanced condition indicates that the other phases have the same current magnitude, but are displaced from each other by 120 electrical degrees.



**Figure 1 Diagram of a Balanced Three-Phase Circuit**

However, the single-phase calculation must take into account the voltage drop across the mutual impedance caused by the other phase currents. Therefore  $Z_{a1}$  is special impedance in which the self-impedance of the line and the mutual impedance between lines are contained as lumped elements for the special condition of balanced current. The special impedance is called the positive-sequence impedance of the line and is derived as follows.



**Figure 2 Self and Mutual Impedance in a Transposed Line**

Figure 2 is the diagram of a transposed transmission line showing the self-impedance  $Z_s$  of each conductor and equal mutual impedance between each pair of conductors. We can write the voltage drop in the A-phase as:

$$V_a = Z_s I_a + Z_m I_b + Z_m I_c = Z_s I_a + Z_m (I_b + I_c) \quad (1)$$

For the special case of balanced three-phase current  $(I_b + I_c) = -I_a$ . Consequently:

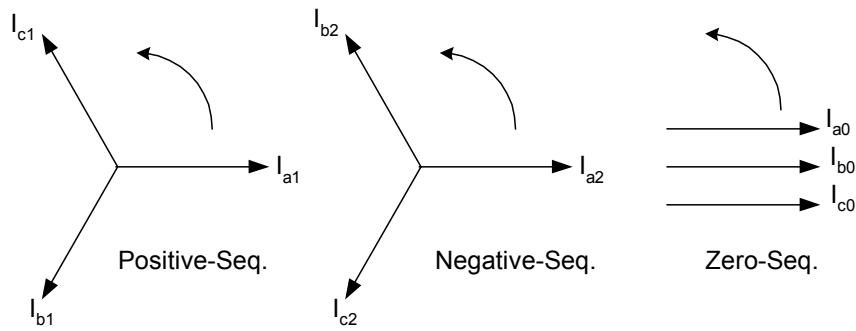
$$V_a = (Z_s - Z_m)I_a \quad (2)$$

Dividing both sides of equation (2) by  $I_a$  shows that the positive-sequence impedance of the line is equal to the self-impedance of the line minus the mutual impedance:

$$Z_{a1} = \frac{V_a}{I_a} = (Z_s - Z_m) \quad (3)$$

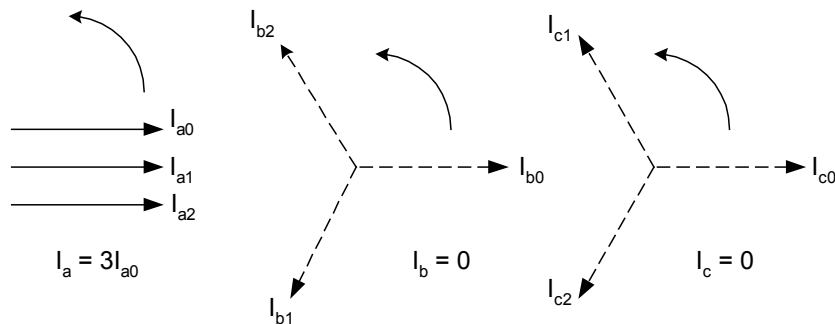
This is the impedance seen by a system of balanced three-phase currents, which are equal in magnitude and displaced by 120 degrees.

By convention, phasors are said to rotate in a counter clockwise direction, and their sequence refers to the order in which three-phase phasors appear passing a stationary point of observation. Three systems of balanced phasors can be defined as shown in Figure 3. Each system has phasors of equal magnitude, but each system has its own unique phase sequence.



**Figure 3 Symmetrical Component Systems (A-Gnd Fault)**

The first is called the positive-sequence system and has the sequence a-b-c generated by the system voltage. The second is called the negative-sequence system and has the sequence a-c-b not generated by the system voltage. The third is called the zero-sequence system because the phasors have the same phase angle and rotate together. It is also a sequence not generated by the system voltage. Collectively, they are the symmetrical components.

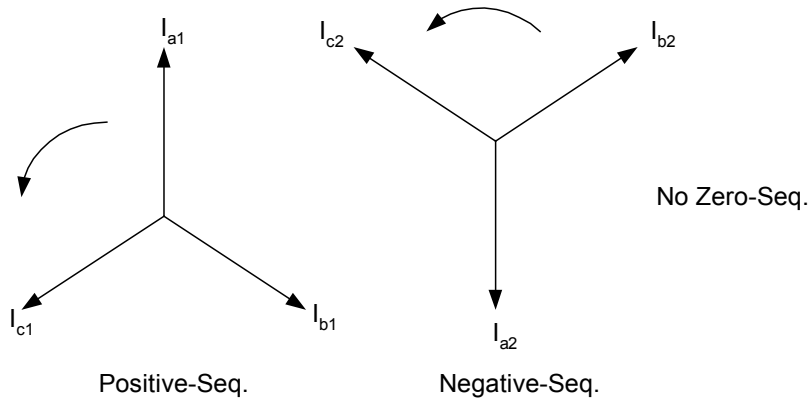


**Figure 4 Adding Symmetrical Components to Obtain Three-Phase Currents**

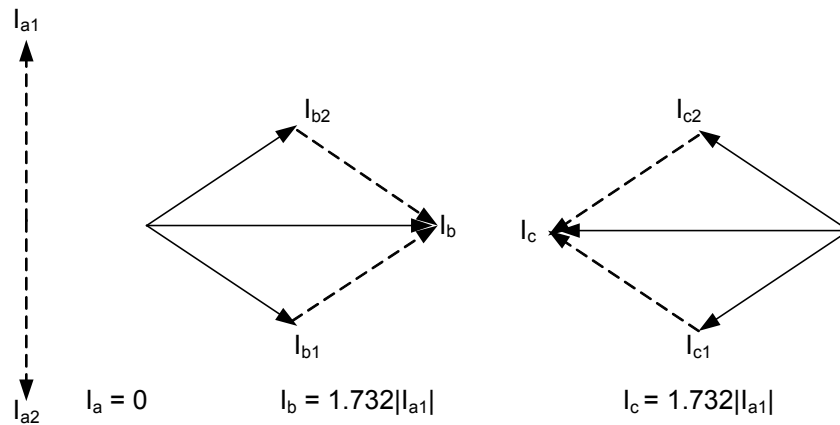
They are significant because, although each is a balanced system, adding them phase-by-phase produces unbalanced phase quantities. Consequently, by choosing the magnitude and reference phase angle of each set, we can use them to represent any degree of phasor unbalance. For

example, the symmetrical components in Figure 3 result from a line-to-ground fault where there is current in Phase A and zero current in B and C. Figure 4 shows the components added phase by phase to reconstitute the phase currents. Since  $I_{a1}$ ,  $I_{a2}$ , and  $I_{a0}$  have the same magnitude and phase angle, the A-phase current equals  $3I_{a0}$ . However, the components for B-phase and C-phase are balanced phasors that add to zero and produce no current.

As a second example, Figure 5 shows the symmetrical components of the phase currents resulting from a B-to-C line-to-line fault. The phase-by-phase addition to reconstitute the phase currents is shown in Figure 6. With no zero-sequence current present, the  $I_{a1}$  and  $I_{a2}$  components cancel to produce zero A-phase current. The positive- and negative- sequence B-phase and C-phase components combine to produce equal and opposite B and C current of the line-to-line fault.



**Figure 5 Symmetrical Components Resulting for a Line-to-Line Fault**



**Figure 6 Adding the Symmetrical Components for a B-to-C Fault**

We can now observe that three-phase currents are composed of symmetrical components that exist simultaneously and independently in the network. Furthermore, each of the components encounters a unique impedance, which incorporates the self, and mutual impedance in a manner determined by its sequence. We have already seen the derivation of the positive-sequence impedance. Because the relation  $(I_b + I_c) = -I_a$ , negative-sequence current encounters a negative-sequence impedance equal to the positive-sequence impedance:

$$Z_{a2} = \frac{V_{a2}}{I_{a2}} = (Z_s - Z_m) \quad (4)$$

However, since the zero-sequence currents  $I_{a0}$ ,  $I_{b0}$ , and  $I_{c0}$  are all in phase, the zero-sequence incorporates the self and mutual impedance as follows. The zero-sequence current flowing in the transposed line shown in Figure 7 causes the voltage drop:

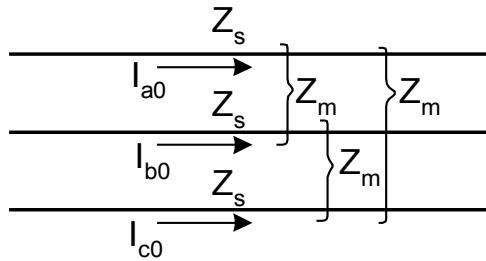
$$V_{a0} = Z_s I_{a0} + Z_m I_{b0} + Z_m I_{c0} = Z_s I_{a0} + Z_m (I_{b0} + I_{c0}) \quad (5)$$

For the special balanced case of zero-sequence current  $I_{b0} + I_{c0} = 2I_{a0}$ . Consequently, the voltage drop can be written as:

$$V_{a0} = (Z_s + 2Z_m) I_{a0} \quad (6)$$

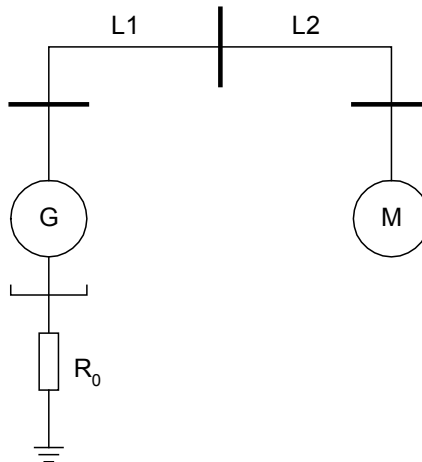
Dividing both sides of Equation 6 by  $I_{a0}$  gives the zero-sequence impedance:

$$Z_{a0} = \frac{V_{a0}}{I_{a0}} = (Z_s + 2Z_m) \quad (7)$$

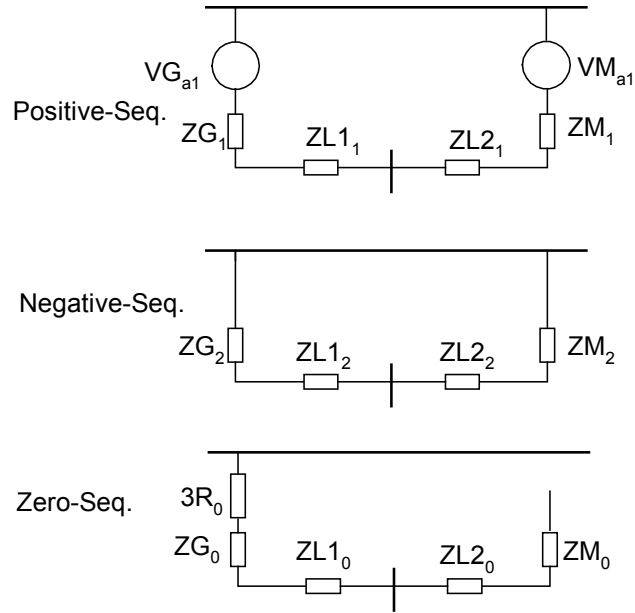


**Figure 7 Zero-Sequence Current in a Transposed Line**

By superposition, we can consider the symmetrical components as flowing in three separate networks called the positive-, negative-, and zero-sequence. Figure 8 shows the one-line diagram of a generator  $G$  feeding a motor  $M$  through a two-section transmission line. The generator is resistance grounded through resistor  $R_0$ , while the motor remains ungrounded. Figure 9 shows the positive-, negative-, and zero-sequence networks of the system.



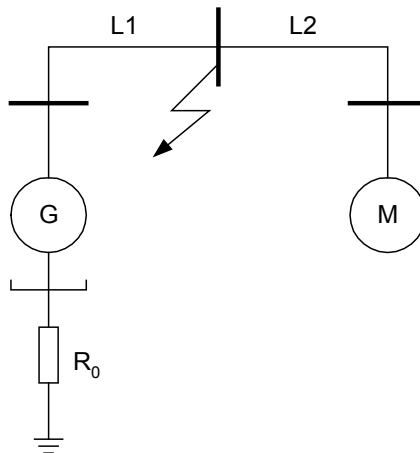
**Figure 8 System One-Line Diagram**



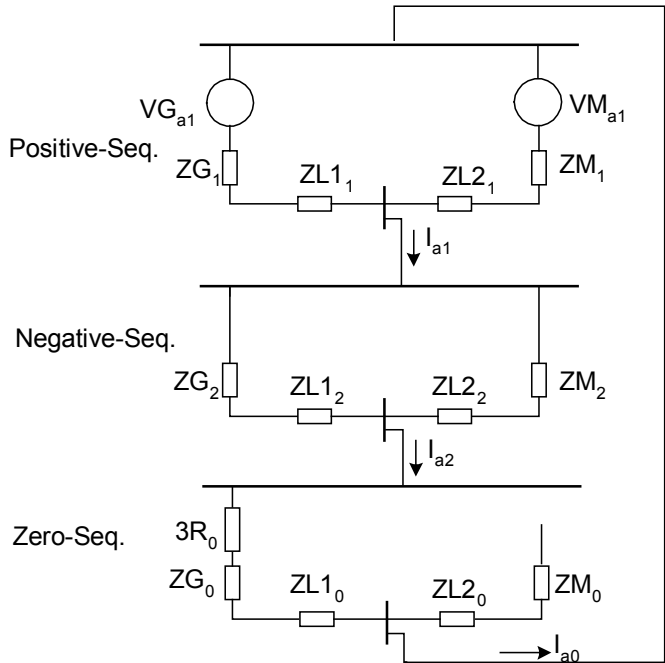
**Figure 9 Symmetrical Component Networks**

The positive-sequence network contains the positive-sequence voltage of the generator, the internal voltage of the motor, and the positive-sequence impedance of the generator, motor, and lines. The negative- and zero-sequence networks contain only sequence impedance.

In Figure 9, there is no generated negative- or zero-sequence voltage and no connection between the networks. Therefore, the diagram shows the normal balanced operating state of the system where only positive-sequence current flows, and there is no negative- or zero-sequence current. The presence of negative- and zero-sequence current indicates an unbalanced fault condition and a connection between networks. We have already seen in Figure 3 that equal current in the A-phase of each network characterizes the Phase A-to-ground fault. Connecting the networks in series at the point of fault produces this condition. The location of the fault is shown in Figure 10, and the connection of the sequence networks is shown in Figure 11. Note that because the motor is ungrounded, all the zero-sequence current flows in the grounding resistor. A resistance of  $3R_0$  is used to produce the same voltage drop as the total ground current  $3I_0$  flowing in the ground resistor.

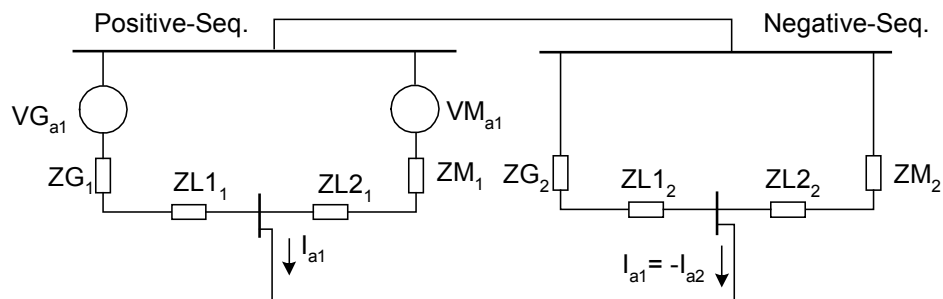


**Figure 10 A-Phase Fault Location**



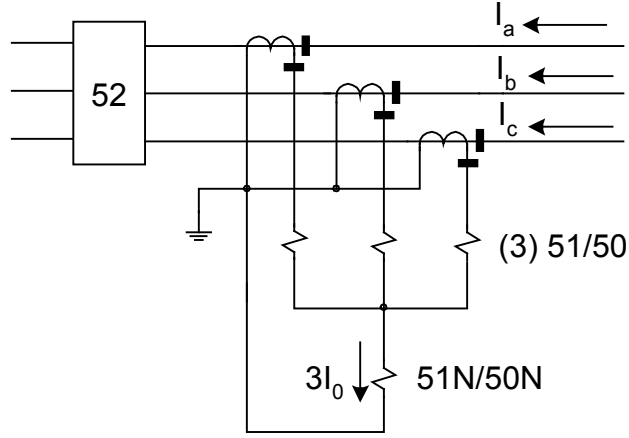
**Figure 11 Sequence Network Connection for Bus 2 A-to-Ground Fault**

We have seen in Figure 5 that only positive- and negative-sequence current are present in a line-to-line fault. Furthermore, we observe that a B-to-C fault is characterized by the fact that  $I_{a1} = -I_{a2}$ . Consequently, the B-to-C fault is represented by a parallel connection between the positive- and negative-sequence networks at the point of fault. The connection is shown in Figure 12.



**Figure 12 Sequence Network Connection for a B-to-C Fault**

Symmetrical components are an essential means for analyzing fault conditions in power systems, and are routinely measured and used as operating quantities in protective relays. For example, neglecting load unbalance, zero-sequence current occurs with ground faults and does not occur with three-phase or line-to-line faults. Figure 4 shows us that the sum of the three-phase currents is equal to  $3I_0$ . Consequently,  $3I_0$  is routinely measured using an overcurrent in the familiar residual connection shown in Figure 13.



**Figure 13 Phase and Residual Overcurrent Relays**

The equation for zero-sequence current is:

$$3I_{a0} = I_a + I_b + I_c \quad (8)$$

$$I_{a0} = \frac{1}{3}(I_a + I_b + I_c) \quad (9)$$

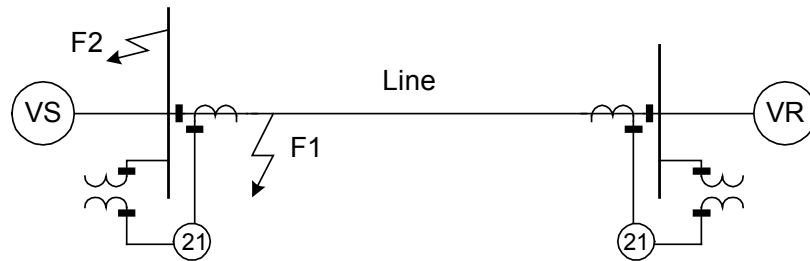
The positive- and negative-sequence components are obtained by applying either a 120- or a 240-degree phase shift to the B and C phasors and then adding the three-phase currents. Let  $a = 1.0 \angle 120^\circ$  and  $a^2 = 1.0 \angle 240^\circ$  be the unity phasors that apply the appropriate phase shifts. Then the equation for positive-sequence current is:

$$3I_{a1} = (I_a + aI_b + a^2I_c) \quad (10)$$

The equation for negative-sequence current is:

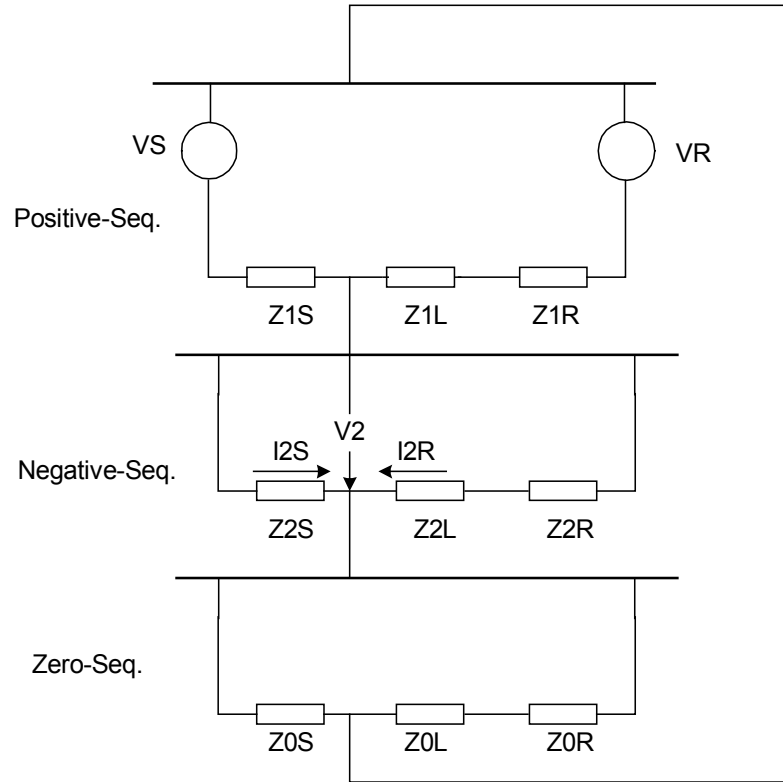
$$3I_{a2} = (I_a + a^2I_b + aI_c) \quad (11)$$

A prime example of the application of symmetrical components is a unique directional element that calculates the negative-sequence impedance at the relay point and determines the fault direction from the magnitude and sign of the calculated negative-sequence impedance. In Figure 14, the relay at Source S must trip for the line-to-ground fault in front of the relay at F1 and restrain for a fault behind the relay at F2. The sequence networks connected for a line-to-ground fault are shown in Figure 15.



**Figure 14 Forward and Reverse Faults**





**Figure 15 Sequence Networks for a Line-to-Ground Fault**

Since there are no sources in the negative-sequence network, the negative-sequence voltage  $V_2$  is the voltage drop across the S-bus source impedance  $Z_{2S}$  caused by the current  $I_{2S}$ .  $V_2$  is also the voltage drop across the impedance  $(Z_{2L} + Z_{2R})$  caused by the current  $I_{2R}$ . If the fault is in front of the relay, its voltage is  $-V_{2S}$  and its current is  $I_{2S}$ . Consequently, the negative-sequence element measures the impedance  $(-Z_{2S})$ . However, if the fault is moved in back of the relay, the current changes abruptly to  $(-I_{2R})$ , and the directional element measures the impedance  $(Z_{2L} + Z_{2R})$ . An impedance threshold, set equal to one-half the line impedance, is an adequate setting. The element asserts for an impedance measurement less than the setting threshold and restrains for a measurement greater than the setting.

## LINE IMPEDANCE MATRIX

Symmetrical components are derived from the impedance matrix of a transposed transmission line. The impedance matrix is obtained by writing equations for the voltage drop due to current in each phase:

$$\begin{aligned}
 V_a &= Z_{aa}I_a + Z_{ab}I_b + Z_{ac}I_c \\
 V_b &= Z_{ba}I_a + Z_{bb}I_b + Z_{bc}I_c \\
 V_c &= Z_{ca}I_a + Z_{cb}I_b + Z_{cc}I_c
 \end{aligned} \tag{1}$$

When using matrix notation the impedance is seen as a 3-by-3 matrix of self and mutual impedance terms and the voltage and current are seen as 1-by-3 column vectors:

$$\begin{pmatrix} \mathbf{V}_a \\ \mathbf{V}_b \\ \mathbf{V}_c \end{pmatrix} = \begin{pmatrix} \mathbf{Z}_{aa} & \mathbf{Z}_{ab} & \mathbf{Z}_{ac} \\ \mathbf{Z}_{ba} & \mathbf{Z}_{bb} & \mathbf{Z}_{bc} \\ \mathbf{Z}_{ca} & \mathbf{Z}_{cb} & \mathbf{Z}_{cc} \end{pmatrix} \begin{pmatrix} \mathbf{I}_a \\ \mathbf{I}_b \\ \mathbf{I}_c \end{pmatrix} \quad (2)$$

Equation (2) can be written in more compact notation where bold letters indicate matrices:

$$\mathbf{V} = \mathbf{Z} \mathbf{I} \quad (3)$$

## SYMMETRICAL COMPONENT MATRIX

The symmetrical component equations for voltage are:

$$\begin{aligned} \mathbf{V}_{a0} &= \frac{1}{3}(\mathbf{V}_a + \mathbf{V}_b + \mathbf{V}_c) \\ \mathbf{V}_{a1} &= \frac{1}{3}(\mathbf{V}_a + a\mathbf{V}_b + a^2\mathbf{V}_c) \\ \mathbf{V}_{a2} &= \frac{1}{3}(\mathbf{V}_a + a^2\mathbf{V}_b + a\mathbf{V}_c) \end{aligned} \quad (4)$$

Which when written in matrix notation appears as:

$$\begin{pmatrix} \mathbf{V}_{a0} \\ \mathbf{V}_{a1} \\ \mathbf{V}_{a2} \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{pmatrix} \begin{pmatrix} \mathbf{V}_a \\ \mathbf{V}_b \\ \mathbf{V}_c \end{pmatrix} \quad (5)$$

Using the more compact notation:

$$\mathbf{V}_{\text{sym}} = \mathbf{A} \mathbf{V} \quad (6)$$

Matrix **A** contains the vector 1 and the plus and minus 120° rotation unity vectors *a* and *a*<sup>2</sup>. Similarly, the symmetrical component equations for current are:

$$\begin{aligned} \mathbf{I}_{a0} &= \frac{1}{3}(\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c) \\ \mathbf{I}_{a1} &= \frac{1}{3}(\mathbf{I}_a + a\mathbf{I}_b + a^2\mathbf{I}_c) \\ \mathbf{I}_{a2} &= \frac{1}{3}(\mathbf{I}_a + a^2\mathbf{I}_b + a\mathbf{I}_c) \end{aligned} \quad (7)$$

Which can be written in matrix form as:

$$\begin{pmatrix} \mathbf{I}_{a0} \\ \mathbf{I}_{a1} \\ \mathbf{I}_{a2} \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{pmatrix} \begin{pmatrix} \mathbf{I}_a \\ \mathbf{I}_b \\ \mathbf{I}_c \end{pmatrix} \quad (8)$$

In more compact notation:

$$\mathbf{I}_{\text{sym}} = \mathbf{A} \mathbf{I} \quad (9)$$

Matrix equations (6) and (9) are the 1-by-3 voltage and current column vectors in terms of the symmetrical component operator  $\mathbf{A}$ . The relation between the symmetrical components and the impedance matrix is obtained by substituting these equations for  $\mathbf{V}$  and  $\mathbf{I}$  in (3):

$$\begin{aligned}\mathbf{A}^{-1} \mathbf{V}_{\text{sym}} &= \mathbf{Z} \mathbf{A}^{-1} \mathbf{I}_{\text{sym}} \\ \mathbf{V}_{\text{sym}} &= \mathbf{A} \mathbf{Z} \mathbf{A}^{-1} \mathbf{I}_{\text{sym}}\end{aligned}\quad (10)$$

Observing the position of terms in (10) we can write the symmetrical component impedance matrix as:

$$\mathbf{V}_{\text{sym}} = \mathbf{Z}_{\text{syn}} \mathbf{I}_{\text{sym}} \quad (11)$$

where the symmetrical component impedance matrix is:

$$\mathbf{Z}_{\text{syn}} = \mathbf{A} \mathbf{Z} \mathbf{A}^{-1} \quad (12)$$

and where  $\mathbf{Z}$  is the transmission line impedance matrix:

$$\mathbf{Z} = \begin{pmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ba} & Z_{bb} & Z_{bc} \\ Z_{ca} & Z_{cb} & Z_{cc} \end{pmatrix} \quad (13)$$

In the  $\mathbf{Z}$  matrix the self impedance terms can be equal only for a flat line configuration such that:

$$Z_s = Z_{aa} = Z_{bb} = Z_{cc} \quad (14)$$

For a transposed line the off-diagonal terms which are the mutual impedances between conductors are equal such that:

$$Z_m = Z_{ab} = Z_{bc} = Z_{ca} \quad (15)$$

The impedance matrix can then be written as:

$$\mathbf{Z} = \begin{pmatrix} Z_s & Z_m & Z_m \\ Z_m & Z_s & Z_m \\ Z_m & Z_m & Z_s \end{pmatrix} \quad (16)$$

The component matrix for the transposed transmission line is:

$$\mathbf{Z}_{\text{syn}} = \mathbf{A} \mathbf{Z} \mathbf{A}^{-1} = \begin{pmatrix} Z_s + 2Z_m & 0 & 0 \\ 0 & Z_s - Z_m & 0 \\ 0 & 0 & Z_s - Z_m \end{pmatrix} \quad (17)$$

In the symmetrical component matrix, the diagonal terms are the zero-sequence, the positive-sequence, and the negative sequence impedance of the transmission line. Note that the off-diagonal terms are zero and indicate that there is no coupling between the positive-, negative-, and zero-sequence networks for the transposed line.

$$\begin{aligned}Z_0 &= Z_s + 2Z_m \\ Z_1 &= Z_s - Z_m \\ Z_2 &= Z_s - Z_m\end{aligned}\quad (18)$$

The impedance matrix for a transposed 100-mile, 500 kV, flat construction line is given in (19). The impedance is given in per unit on a 500 kV, 1000 MVA base:

$$\mathbf{Z} = \begin{pmatrix} 0.048 + j0.432 & 0.036 + j0.17 & 0.036 + j0.17 \\ 0.036 + j0.17 & 0.048 + j0.432 & 0.036 + j0.17 \\ 0.036 + j0.17 & 0.036 + j0.17 & 0.048 + j0.432 \end{pmatrix} \quad (19)$$

The symmetrical component matrix is obtained by applying equation (12):

$$\mathbf{Z}_{\text{syn}} = \mathbf{A} \mathbf{Z} \mathbf{A}^{-1} = \begin{pmatrix} 0.12 + j0.771 & 0 & 0 \\ 0 & 0.012 + j0.263 & 0 \\ 0 & 0 & 0.012 + j0.263 \end{pmatrix} \quad (20)$$

Where the zero-sequence, positive-sequence, and negative-sequence impedance terms are:

$$\begin{aligned} Z_0 &= 0.12 + j0.771 \\ Z_1 &= 0.012 + j0.262 \\ Z_2 &= 0.012 + j0.262 \end{aligned} \quad (21)$$

## TRANSPPOSED LINES

The three-phase fault calculations for a transposed line are as follows:

$$\begin{pmatrix} \mathbf{V}_a \\ \mathbf{V}_b \\ \mathbf{V}_c \end{pmatrix} = \begin{pmatrix} 1 \\ \mathbf{a}^2 \\ \mathbf{a} \end{pmatrix} \quad \begin{pmatrix} \mathbf{I}_a \\ \mathbf{I}_b \\ \mathbf{I}_c \end{pmatrix} = \mathbf{Z}^{-1} \begin{pmatrix} \mathbf{V}_a \\ \mathbf{V}_b \\ \mathbf{V}_c \end{pmatrix} \quad \begin{pmatrix} \mathbf{I}_a \\ \mathbf{I}_b \\ \mathbf{I}_c \end{pmatrix} = \begin{pmatrix} 0.175 - j3.803 \\ -3.381 + j1.75 \\ 3.206 + j2.053 \end{pmatrix}$$

$$Z_{1a} = \frac{\mathbf{V}_a - \mathbf{V}_b}{\mathbf{I}_a - \mathbf{I}_b} \quad Z_{1b} = \frac{\mathbf{V}_b - \mathbf{V}_c}{\mathbf{I}_b - \mathbf{I}_c} \quad Z_{1c} = \frac{\mathbf{V}_c - \mathbf{V}_a}{\mathbf{I}_c - \mathbf{I}_a}$$

$$\left| \frac{Z_{1a}}{Z_{a1}} \right| = 1.00 \quad \left| \frac{Z_{1b}}{Z_{a1}} \right| = 1.00 \quad \left| \frac{Z_{1c}}{Z_{a1}} \right| = 1.00$$

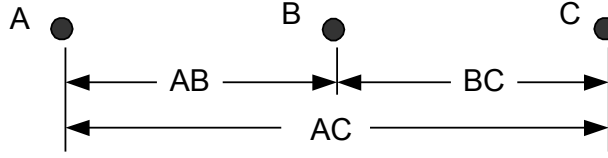
The three-phase calculation using symmetrical components gives:

$$\begin{pmatrix} \mathbf{V}_{a0} \\ \mathbf{V}_{a1} \\ \mathbf{V}_{a2} \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad \begin{pmatrix} \mathbf{I}_{a0} \\ \mathbf{I}_{a1} \\ \mathbf{I}_{a2} \end{pmatrix} = \mathbf{Z}_{\text{sym}}^{-1} \begin{pmatrix} \mathbf{V}_{a0} \\ \mathbf{V}_{a1} \\ \mathbf{V}_{a3} \end{pmatrix} \quad \begin{pmatrix} \mathbf{I}_{a0} \\ \mathbf{I}_{a1} \\ \mathbf{I}_{a2} \end{pmatrix} \frac{1}{\mathbf{I}_{a1}} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$Z_{a1} = \frac{\mathbf{V}_{a1}}{\mathbf{I}_{a1}} \quad \frac{\text{Im}(Z_{a1})}{\text{Im}(Z_{\text{sym}_{1,1}})} = 1.00$$

The calculation for the transposed line indicates an ideal mho element response. Such is not the case for a nontransposed line.

## NONTRANSPOSED LINES



**Figure 16 Line with Horizontal Construction**

In the configuration shown in Figure 1, the distance AB is the same as BC, and we can expect the mutual impedance  $Z_{ab}$  and  $Z_{bc}$  to be equal. However, because of the wider spacing AC,  $Z_{ac}$  will not have the same value. The matrix for a 100-mile nontransposed 500 kV line with flat construction is given in (22). The impedance is in per unit on a 500 kV 1000 MVA base:

$$Z = \begin{pmatrix} 0.048 + j0.432 & 0.036 + j0.181 & 0.036 + j0.147 \\ 0.036 + j0.181 & 0.048 + j0.432 & 0.036 + j0.181 \\ 0.036 + j0.147 & 0.036 + j0.181 & 0.048 + j0.432 \end{pmatrix} \quad (22)$$

The symmetrical component matrix in this case is:

$$Z_{\text{sym}} = \mathbf{A} \mathbf{Z} \mathbf{A}^{-1} = \begin{pmatrix} 0.12 + j0.771 & 0.01 + j0.006 & 0.01 + j0.006 \\ 0.01 + j0.006 & 0.012 + j0.262 & -0.019 + j0.011 \\ 0.01 + j0.006 & -0.019 + j0.011 & 0.012 + j0.262 \end{pmatrix} \quad (23)$$

The impedance elements in the matrices (22) and (24) are more easily compared in polar form:

$$Z = \begin{pmatrix} 0.434e^{j83.7} & 0.185e^{j78.7} & 0.151e^{j76.2} \\ 0.185e^{j78.7} & 0.434e^{j83.7} & 0.185e^{j78.7} \\ 0.151e^{j76.2} & 0.185e^{j78.7} & 0.434e^{j83.7} \end{pmatrix} \quad (24)$$

$$Z_{\text{sym}} = \mathbf{A} \cdot \mathbf{Z} \cdot \mathbf{A}^{-1} = \begin{pmatrix} 0.780e^{81.2} & 0.012e^{j30.9} & 0.012e^{j30.9} \\ 0.012e^{j30.9} & 0.263e^{j87.4} & 0.023e^{j149.9} \\ 0.012e^{j30.9} & 0.023e^{j149.9} & 0.263e^{j87.4} \end{pmatrix} \quad (25)$$

In this case, the off-diagonal terms are not zero, indicating that there is coupling between the sequence networks for all types of faults. The elements of the symmetrical component matrix are:

$$Z_{\text{sym}} = \begin{pmatrix} Z_{00} & Z_{01} & Z_{02} \\ Z_{10} & Z_{11} & Z_{12} \\ Z_{20} & Z_{21} & Z_{22} \end{pmatrix} \quad (26)$$

where:

- $Z_{00}$  = Zero Sequence Impedance
- $Z_{11}$  = Positive Sequence Impedance
- $Z_{22}$  = Negative Sequence Impedance
- $Z_{21}$  = Mutual Impedance ( $V_{a2}$  drop due to  $I_{a1}$  current)
- $Z_{01}$  = Mutual Impedance ( $V_{a0}$  drop due to  $I_{a1}$  current)

The calculation of a line end three-phase fault shows the effect of nontransposition on relay reach:

$$\begin{pmatrix} V_a \\ V_b \\ V_c \end{pmatrix} = \begin{pmatrix} 1 \\ a^2 \\ a \end{pmatrix} \begin{pmatrix} I_a \\ I_b \\ I_c \end{pmatrix} = Z^{-1} \begin{pmatrix} V_a \\ V_b \\ V_c \end{pmatrix} \quad \begin{pmatrix} I_a \\ I_b \\ I_c \end{pmatrix} = \begin{pmatrix} 0.506 - j3.66 \\ -3.652 + j1.88 \\ 3.317 + j1.725 \end{pmatrix}$$

$$Z_{1a} = \frac{V_a - V_b}{I_a - I_b} \quad Z_{1b} = \frac{V_b - V_c}{I_b - I_c} \quad Z_{1c} = \frac{V_c - V_a}{I_c - I_a}$$

$$\left| \frac{Z_{1a}}{Z_{a1}} \right| = 0.952 \quad \left| \frac{Z_{1b}}{Z_{a1}} \right| = 0.946 \quad \left| \frac{Z_{1c}}{Z_{a1}} \right| = 1.085$$

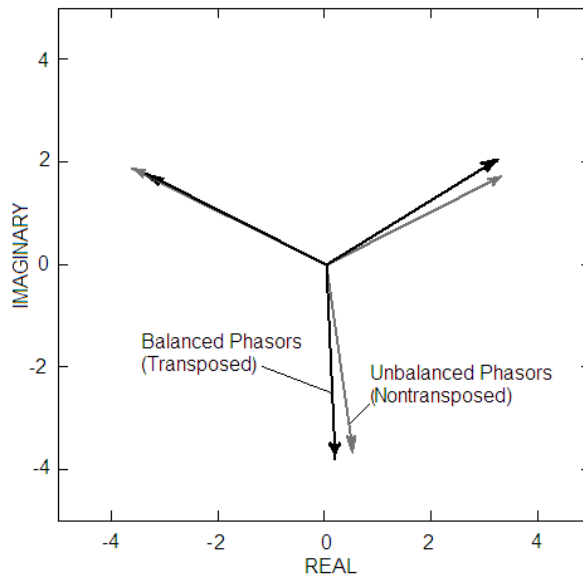
The calculation shows that the phase A and phase B who elements underreach by 4.8% and 5.4% respectively, while the phase C who element overreaches by 8.5%. Consequently, the relay settings must take into account the effect of nontransposition.

The three-phase fault calculation using symmetrical components gives

$$\begin{pmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{pmatrix} = Z_{\text{sym}}^{-1} \begin{pmatrix} V_{a0} \\ V_{a1} \\ V_{a3} \end{pmatrix} \quad \begin{pmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{pmatrix} \frac{1}{I_{a1}} = \begin{pmatrix} 0.006 + j0.015 \\ 1 \\ 0.046 + j0.072 \end{pmatrix}$$

$$Z_{a1} = \frac{V_{a1}}{I_{a1}} \quad \frac{\text{Im}(Z_{a1})}{\text{Im}(Z_{\text{sym}_{1,1}})} = 0.992$$

Consequently, a negative-sequence current of 8.6% of the three-phase fault current and a 4.7% residual current are produced by a fault on the nontransposed line.



**Figure 17 Three-Phase Fault Unbalanced Currents**

The unbalanced phase currents produced by three-phase fault on the nontransposed line are shown in Figure 17. The phase currents and corresponding symmetrical components are:

$$\mathbf{I} = \begin{pmatrix} 3.96e^{j(81.09)} \\ 4.113e^{j152.75} \\ 3.74e^{j37.39} \end{pmatrix} \quad \mathbf{I}_{\text{sym}} = \mathbf{A} \cdot \mathbf{I} = \begin{pmatrix} 0.182e^{j(-18.28)} \\ 11.52e^{j(-87.33)} \\ 1.00e^{j35.36} \end{pmatrix}$$

M. H. Hess [3] shows how the mutual impedance terms of the symmetrical impedance matrix can be used to estimate the negative- and zero-sequence current during a three-phase fault:

$$I_{a2} = \frac{|Z_{21}|}{|Z_{22}|} I_{a1} = \frac{0.23}{0.263} 11.52 = 1.00 \quad I_{a0} = \frac{|Z_{01}|}{|Z_{00}|} I_{a1} = \frac{.012}{0.780} 11.52 = 0.167$$

## CONCLUSIONS

1. Symmetrical components are a practical tool for analyzing unbalanced conditions on balanced systems.
2. Real systems are unbalanced.
3. Balanced faults can produce unbalanced currents and voltages.
4. Be careful when setting sensitive negative-sequence and ground elements.

## REFERENCES

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## BIOGRAPHY

**Dr. Edmund O. Schweitzer, III** is recognized as a pioneer in digital protection, and holds the grade of Fellow of the IEEE, a title bestowed on less than one percent of IEEE members. In 2002 he was elected a member of the National Academy of Engineering. He is the recipient of the Graduate Alumni Achievement Award from Washington State University and the Purdue University Outstanding Electrical and Computer Engineer Award. He has written dozens of technical papers in the areas of digital relay design and reliability and holds more than 20 patents pertaining to electric power system protection, metering, monitoring, and control. Dr. Schweitzer received his Bachelor's degree and his Master's in electrical engineering from Purdue University, and his Ph.D. degree from Washington State University. He served on the electrical engineering faculties of Ohio University and Washington State University, and in 1982 he founded Schweitzer Engineering Laboratories to develop and manufacture digital protective relays and related products and services.

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His biography appears in Who's Who in America. He holds over a dozen patents associated with power system protection using solid state and microprocessor technology and is the author of numerous IEEE and Protective Relay Conference papers. He received the Best Paper Award of the 1988 Petroleum and Chemical Industry Conference and the Power System Relaying Committee's Distinguished Service Award in 1991.