Superimposed Quantities: Their True Nature and Application in Relays

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1. INTRODUCTION

The idea of ultra-high-speed directional relays was first conceived in the late nineteen-seventies [1-4]. There has been confusion between relays based on superimposed quantities and relays based on traveling waves. In early publications, both principles were discussed at the same time. This is because relays based on traveling waves use the superimposed voltages and currents to assess the changes occurring on the line [5-6]. Equally, few details were disclosed in the beginning about the practical aspects of the algorithms. More recently, two papers [7-8] discuss the use of superimposed quantities to assess fault direction. For the applications discussed in these papers, there was no harsh timing requirement. This allowed using comparators processed in the frequency domain with the conventional use of phasors.

The purpose of this paper is to show that linear circuit theory and conventional comparators techniques can fully characterize the performance (speed, advantages, and shortcomings) of directional elements based on time-varying waveforms or their phasors. In addition, we show that superimposed quantities are suitable for fault-type selection given the proper design and supervision considerations.

2. DERIVATION OF SUPERIMPOSED QUANTITIES USING THE SUPERPOSITION PRINCIPLE

2.1 Fault Analysis Using the Superposition Principle

Consider the single-line diagram of Figure 1, where a fault is applied through a resistance Rf at a distance m per-unit line length from the relay at the left bus.

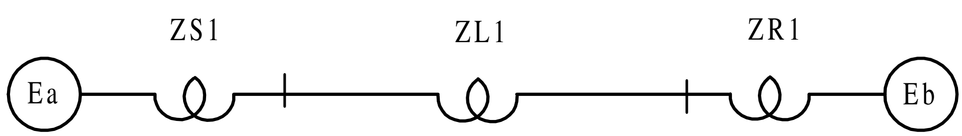

Figure 1: Example System Single-Line Diagram

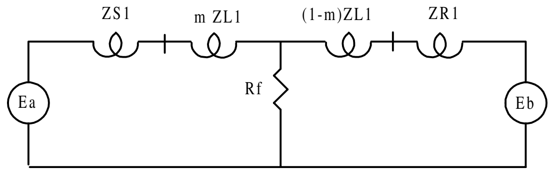

Figure 2a: Faulted Network

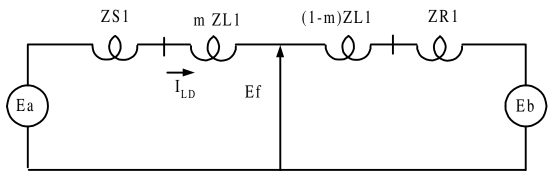

Figure 2b: Pre-Fault Network

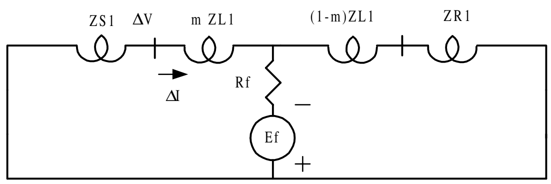

Figure 2c: Pure-Fault Network

The voltage Eb of the right-hand source is expressed as:

$$E_b = h \bullet e^{-j\theta} \bullet E_a \qquad (2.1)$$

where θ is the angular difference between the left and right sources and h is a scalar. For negative values of θ, load flows from left to right.

We can use the superposition principle to determine the voltages and currents of the faulted circuit in the presence of load flow. To accomplish this, we must first define the pre-fault (Figure 2b) and pure-fault networks (Figure 2c).

The pure-fault network is defined as follows:

a) The pre-fault network voltage sources must be short-circuited
b) A voltage source, Ef, must be applied at the fault point.

The magnitude of Ef is equal to the voltage level existing at the fault location before application of the fault. The source phase angle is opposite to that of the pre-fault voltage phase angle at the fault point.

Determine either a faulted circuit voltage (V) or a current (I) by summing two components, pre-fault plus pure-fault, as provided by the superposition principle (in all equations, capital letters represent phasors, small letters are scalars):

$$V = V_{pre-flt} + \Delta V \qquad (2.2)$$
$$I = I_{pre-flt} + \Delta I$$

The pure-fault network currents and voltages are zero before the fault. Therefore, any value they have due to a fault condition represents a change or delta quantity. For this reason, they are called incremental or superimposed quantities and are represented with a prefix Δ to indicate the change with respect to the pre-fault circuit values.

3. SUPERIMPOSED QUANTITIES FOR CONVENTIONAL SHUNT FAULTS

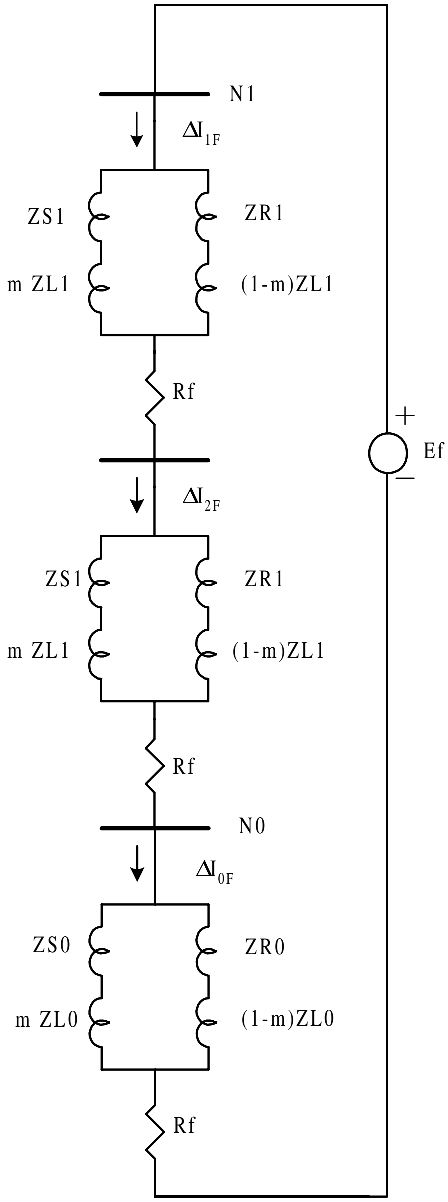

Figure 3: Pure-Fault Sequence Network for a Single-Phase-to-Ground Fault

The circuits shown in Figures 2a, 2b, and 2c represent a three-phase fault and cannot be used to analyze other conventional shunt faults. To investigate different faults, you must use the appropriate sequence network to represent the pure-fault network. Because the sequence network is used to represent the pure-fault network, all sequence quantities are represented as delta quantities.

3.1 Analysis of a Phase A-to-Ground Fault

Figure 3 represents the pure-fault network of a phase A-to-ground fault. Following the circuit of Figure 1, the phase A pre-fault or load current is expressed as:

$$I_{LD} = \frac{E_a \bullet (1 - h \bullet e^{-j\theta})}{ZS1 + ZL1 + ZR1} \tag{3.1}$$

The voltage Ef at the fault point before the fault is applied is given as:

$$Ef = E_a - (ZS1 + m \bullet ZL1) \bullet I_{LD} \tag{3.2}$$

The incremental phase A current, at the relay, is provided by:

$$\Delta I_{AR} = C1 \bullet \Delta I_{1F} + C2 \bullet \Delta I_{2F} + C0 \bullet \Delta I_{0F} \tag{3.3}$$

and therefore, according to the superposition principle, the total phase A current is:

$$I_{AR} = C1 \bullet \Delta I_{1F} + C2 \bullet \Delta I_{2F} + C0 \bullet \Delta I_{0F} + I_{LD} \tag{3.4}$$

In these expressions, C1, C2, and C0 are the current distribution factors [9].

Perform the same analysis to calculate the voltage at the relay. The phase A pre-fault voltage is:

$$V_{AR(pre\_flt)} = m \bullet ZL1 \bullet I_{LD} + Ef \tag{3.5}$$

The incremental phase A voltage at the relay is:

$$\Delta V_{AR} = -C1 \bullet \Delta I_{1F} \bullet ZS1 - C2 \bullet \Delta I_{2F} \bullet ZS1 - C0 \bullet \Delta I_{0F} \bullet ZS0 \tag{3.6}$$

Using the superposition principle, the fault voltage at the relay is:

$$V_{AR} = -2\,C_1 \bullet \Delta I_{1F} \bullet ZS1 - C_0 \bullet \Delta I_{0F} \bullet ZS0 + Ef + m \bullet ZL1 \bullet I_{LD} \tag{3.7}$$

The principles used in this analysis are easily extended to other types of shunt faults as double-phase and double-phase-to-ground faults by replacing the fault sequence network with the appropriate sequence network for the fault type of interest.

3.2 Definition of an Incremental Impedance

We define an incremental impedance as the ratio of an incremental voltage phasor divided by an incremental current phasor. The incremental impedance can be single-phase A, B, or C, or it can be a differential with the voltage (and current) being taken between two phases (AB, BC, or CA). Finally, it can be computed with incremental sequence quantities. As an example, the phase A incremental impedance, as measured at the relay, for a phase A-to-ground fault is given as:

$$\frac{\Delta V_{AR}}{\Delta I_{AR}} = \Delta Z_{AR} = \frac{-(C1 \bullet ZS1 \bullet \Delta I_{1F} + C2 \bullet ZS1 \bullet \Delta I_{2F} + C0 \bullet ZS0 \bullet \Delta I_{0F})}{C1 \bullet \Delta I_{1F} + C2 \bullet \Delta I_{1F} + C0 \bullet \Delta I_{0F}} \quad (3.8)$$

or:

$$\frac{\Delta V_{AR}}{\Delta I_{AR}} = \Delta Z_{AR} = \frac{-(2 \bullet C1 \bullet ZS1 + C0 \bullet ZS0)}{C1 + C2 + C0} \quad (3.9)$$

The positive-sequence impedance at the relay for a phase A-to-ground fault is provided by:

$$\Delta Z1_R = \frac{\Delta V1_R}{\Delta I1_R} = \frac{-C1 \bullet ZS1 \bullet \Delta I_{1F}}{C1 \bullet \Delta I_{1F}} = -ZS1 \quad (3.10)$$

The incremental impedance across phases A and B is provided by:

$$\Delta ZAB_R = \frac{\Delta(VA - VB)_R}{\Delta(IA - IB)_R} = \frac{-2 \bullet C1 \bullet ZS1 \bullet \Delta I_{1F} \bullet (1 - a^2)}{2 \bullet C1 \bullet \Delta I_{1F} \bullet (1 - a^2)} = -ZS1 \quad (3.11)$$

In Equation 3.11, a is the operator equal to $1\angle 120°$. Notice that the incremental impedance across two phases (one of them being phase A) or using the positive-sequence quantities is equal to the negative of the source impedance behind the relay.

3.3 Incremental Impedances for Other Types of Shunt Faults

In the previous section, we showed that for a single-phase-to-ground fault, properly selected incremental impedances equaled the negative of the source impedance behind the relay. The same principle applies for the other types of shunt faults. Table 1 lists the incremental impedances equal to the negative of the source impedance for the four basic fault types.

Table 1. Incremental Impedances Being Equal to –ZS1

Fault Type	Incremental Impedances
A-G	ΔZab, ΔZca, ΔZ1
B-C	ΔZb, ΔZc, ΔZab, ΔZbc, ΔZca, ΔZ1
BC-G	ΔZab, ΔZbc, ΔZca, ΔZ1
ABC	ΔZa, ΔZb, ΔZc, ΔZab, ΔZbc, ΔZca, ΔZ1

It is interesting to note that the incremental impedances computed across two phases and the positive-sequence impedance are always equal to –ZS1 for all fault types.

4. RELATION BETWEEN SUPERIMPOSED QUANTITIES AND SEQUENCE QUANTITIES

Looking at the pure-fault sequence network of Figure 3, all sequence voltages and currents are represented as superimposed quantities. Sequence quantities are, however, normally computed based on the measured fault voltages and currents. For instance, the pure-fault positive-sequence current at the fault is provided as:

$$\Delta I_{1F} = \Delta I_a + a \bullet \Delta I_b + a^2 \bullet \Delta I_c \tag{4.1}$$

Normally we would compute the positive-sequence current as:

$$I_{1F} = I_a + a \bullet I_b + a^2 \bullet I_c \tag{4.2}$$

Given that any phase current is equal to the pure-fault phase current plus the load:

$$I_{AF} = \Delta I_{AF} + I_{LD} \tag{4.3}$$

we end up with the relation that the computed positive sequence current and the pure-fault positive-sequence current are different by a quantity equal to the load:

$$I_{1F} = \Delta I_{1F} + I_{LD} \tag{4.4}$$

When we apply the same reasoning to both the negative- and zero-sequence currents, the load current vanishes if we assume it to be a balanced quantity. For these two sequence types, the calculated sequence quantities are equal to the pure-fault quantities:

$$I_{2F} = \Delta I_{2F} \quad \text{and} \quad I_{0F} = \Delta I_{0F} \tag{4.5}$$

In conclusion, with the exception of positive-sequence quantities, the calculated sequence quantities are superimposed quantities.

5. FUNDAMENTAL RELATION BETWEEN SUPERIMPOSED VOLTAGE AND SUPERIMPOSED CURRENT

5.1 Fundamental Directional Equation

As we discussed above, selecting the proper quantities at the relay location for each forward fault-type yields an incremental impedance equal to the negative of the positive-sequence source impedance ZS1:

$$\Delta Z_R = \frac{(\text{post–fault } V_R) - (\text{pre–fault } V_R)}{(\text{post–fault } I_R) - (\text{pre–fault } I_R)} = \frac{\Delta V_R}{\Delta I_R} = -ZS1 \tag{5.1}$$

Alternatively, the same condition can be expressed as:

$$\frac{\Delta V_R}{\Delta I_R \bullet (-ZS1)} = 1 \tag{5.2}$$

Equation 5.2 indicates that during a fault, the magnitude and phase of the incremental voltage waveform (or phasor) are equal to the magnitude and phase of the incremental current waveform (or phasor) multiplied by the negative of the source impedance behind the relay. This principle has been exploited to define a directional element [3, 8, 10, 11]. If the scalar product between the incremental voltage phasor and the incremental current phasor, multiplied by the negative of the source impedance, is positive – a forward fault direction is declared:

$$\mathrm{real}(\Delta V_R \bullet \mathrm{conj}(\Delta I_R \bullet (-ZS1)) = \Delta v_R \bullet \Delta i_R \bullet zs1 \bullet \cos\theta \qquad (5.3)$$

In this expression, θ represents any phase angle mismatch that could exist in the source phase angle representation. Normally θ is equal to zero. The magnitude of the source impedance, being always positive, can be set to unity without affecting the basic principle:

$$\mathrm{real}(\Delta V_R \bullet \mathrm{conj}(\Delta I_R \bullet (\angle -ZS1)) = \Delta v_R \bullet \Delta i_R \bullet \cos\theta \qquad (5.4)$$

If the result of Equation 5.4 is negative, the direction is reverse. Thus, for reverse faults the impedance presented to the relay is the sum of the line impedance plus the remote source impedance.

5.2 Impact of Parallel Lines on the Value of the Source Impedance

In more complex networks, like the double circuit shown in Figure 4, even the positive-sequence incremental impedance fails to exactly measure the source impedance behind the relay for three-phase faults.

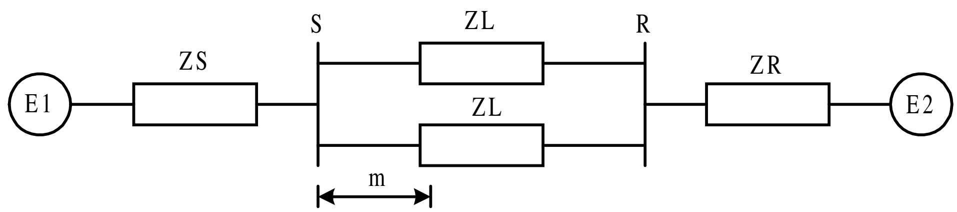

Figure 4: Double Circuit Network

In this case, $\Delta Z1$ is provided by:

$$\Delta Z1 = \frac{-ZS1}{1 + \frac{ZS1}{ZL} - \frac{ZR1}{ZL}\left(\frac{ZS1 + \frac{m}{2}ZL}{ZR1 + \frac{1-m}{2}ZL}\right)} \qquad (5.5)$$

For m = 0, we have

$$\Delta Z1 = \frac{-ZS1 \bullet (2 \bullet ZR1 + ZL)}{(ZS1 + 2 \bullet ZR1 + ZL)} \qquad (5.6)$$

and m = 1:

$$\Delta Z1 = -2 \bullet ZS1 \qquad (5.7)$$

Equations 5.6 and 5.7 indicate that the positive-sequence incremental impedance varies, depending on the location of the fault. The difference in amplitude varies from a small fraction to twice its nominal value.

If the value of the local source impedance varies, it is important the new value remains highly inductive to maintain directionality. Directionality, as provided by Equation 5.3, is still maintained if the mismatch θ remains acceptable. Note that source impedance magnitude variations are not important as it can be set to unity. However, the source impedance magnitude must not be such that the measured current decreases below the sensitivity threshold of the measuring relay.

5.3 Conventional Networks and Exception of Series Compensated Networks

For conventional networks, the source impedance behind a relay is inductive, and applying Equation 5.3 for directionality is applicable without restriction. For series compensated lines, as shown in Figure 5, an adverse situation might develop if the directional relay voltage is supplied from the line side of the capacitors. If the capacitor impedance becomes greater than the original source impedance (ZS1), then the source impedance behind the relay is capacitive and the directional relay makes an incorrect directional declaration.

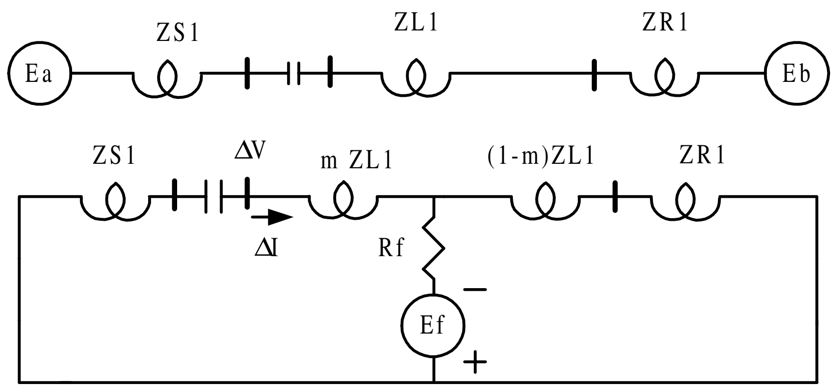

Figure 5: Series Compensated Line

6. Emulation of the Source Impedance Behind a Relay Using a Mimic

6.1 Definition of a Mimic Filter

In Equation 5.3, the incremental current phasor must be multiplied by the negative of the unit source impedance behind the relay to get a compensated current. This can be accomplished in the time domain by processing the current waveform through a high-pass filter, or mimic, of the form [12]:

$$K\,(1 + s \bullet \tau_1) \qquad (6.1)$$

In so doing, we fulfill two objectives:

- multiply the current phasor by the unit source impedance behind the relay.
- remove any dc offset present in the waveform.

Reference [12] shows that the digital form (using the z transform) of the analog high-pass filter expressed by Equation 6.1 is provided by:

$$K\,[(1 + \tau_1) - \tau_1 \bullet z^{-1})] \qquad (6.2)$$

where τ_1 is the filter time constant and K is chosen such that at 60 Hz, the gain is 1.

6.2 Removal of DC Offset by the Mimic Filter

Figure 6 illustrates the removal of a dc offset added to a sine wave after it has been processed through a mimic filter in the time domain. Reference [12] shows that proper removal of any dc offset effect occurs over a large interval of the network X/R ratio.

6.3 Frequency Response of the Mimic

Figure 7 shows the frequency response of the mimic filter. From this figure, notice that the mimic filter is a high-pass filter. While the mimic filter does remove dc from the original waveform, the higher frequency components (if they exist) are amplified.

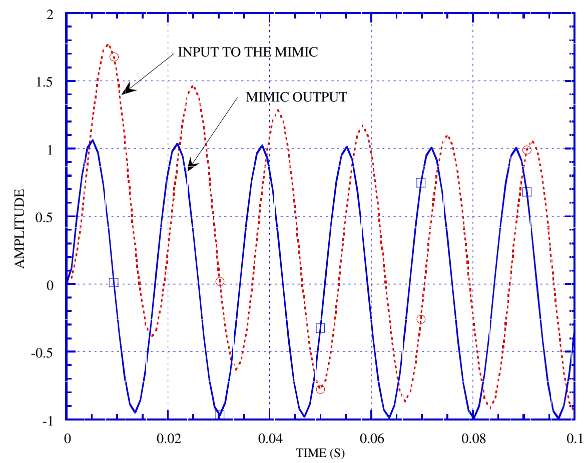

Figure 6: Mimic Filter Removes DC Offset

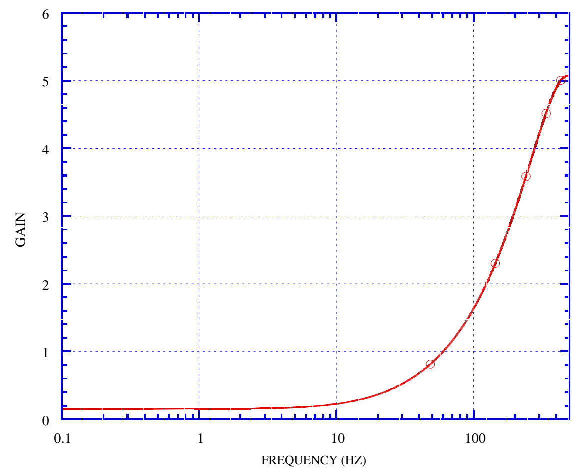

Figure 7: Mimic Frequency Response Passes High Frequencies

6.4 Relationship Between the Mimic, Line Impedance, and Source Impedance Behind the Relay

When we implement the mho type fault detector, we must compute two voltage phasors: the operating voltage and the polarizing voltage [9, 13-15]:

$$S_{op} = I_r \bullet k \bullet ZL1 - V_r \quad (6.3)$$

$$S_{pol} = V_r \quad (6.4)$$

In these two equations, we have:

V_r = particular loop voltage phasor

I_r = particular loop current phasor

$k \bullet ZL1$ = reach of the mho element

In this example, we show the simpler case of a self-polarizing mho element. When computing the operating voltage, we must multiply the loop current by the positive-sequence line impedance scaled by the reach setting. This can be done in the frequency domain as shown in Equation 6.4 by multiplying two complex numbers. It can also be performed in the time domain by equating the phase angle of the mimic of the preceding section to the phase angle of the line and processing the current waveform through the high-pass filter. Next, multiply the phasor of the replica line impedance compensated current by the magnitude of the line reach. The advantage of this technique is that any dc offset is automatically removed. Equation 6.3 then becomes:

$$S_{op} = I_r \bullet (1\angle\theta_{ZL1}) \bullet k \bullet zl1 - V_r \qquad (6.5)$$

Letting the positive-sequence line impedance equal:

$$ZL1 = zl1 \angle\theta_{ZL1} \qquad (6.5)$$

we would then have:

$$\text{arctg}\,(w \bullet \tau_1) = \theta_{ZL1} \qquad (6.6)$$

The same compensated current can now be used to compute the scalar product of Equation 5.4 necessary for assessing directionality of the fault. In doing this, the local source phase angle is equated to the line angle. If both the protected line and source are inductive, even a large mismatch between these angles does not adversely effect the directionality. In theory, the mismatch θ could be as high as 90° before changing the sign of the scalar product.

In a practical digital relay design, the high-pass filter corresponding to Equation 6.2 processes all three phase currents after the relay converts the currents to digital quantities. Then, any algorithm for phasor computation is applied and the compensated current phasors are available for any further processing.

7. MEASUREMENT OF SUPERIMPOSED VOLTAGES AND CURRENTS

7.1 Definition of a Delta-Filter and its Properties

The conventional circuit used for the purpose of extracting a superimposed quantity is known as a delta-filter and is represented in Figure 8. The basic delta-filter subtracts from a time waveform the same waveform delayed by an integral number times the waveform period.

In a delta-filter, the delayed waveform is called the reference signal**.** The delay implemented in the filter is called the delta-filter delay.

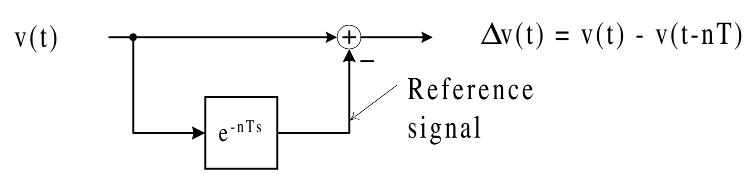

Figure 8: **Concept of a Delta-Filter for a Time-Varying Waveform**

7.2 Frequency Response and Time-Response to Step-Function of a Delta-Filter

A delta-filter is a time-invariant linear filter. Figure 9 shows the frequency response of a delta-filter with a delay corresponding to one 60-Hz period. This plot, however, is misleading because you might conclude that a delta-filter rejects the 60-Hz fundamental component and the harmonics. The filter response to a unit-step 60-Hz sine wave is more revealing (see Figure 10). This figure shows that the filter output over an interval of time equal to one period is equal to the change impressed on the input waveform. In this case, the change is a unit 60-Hz period because the waveform originally did not exist.

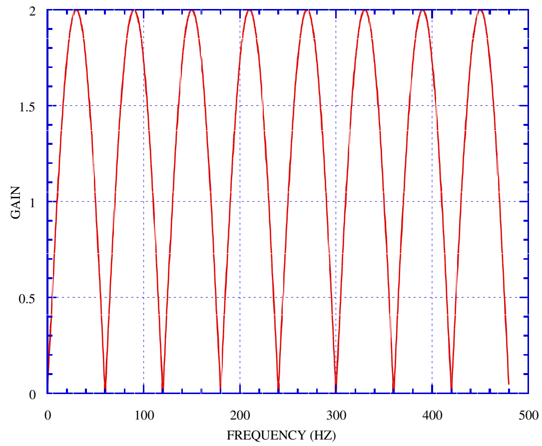

Figure 9: Frequency Response of a Delta-Filter

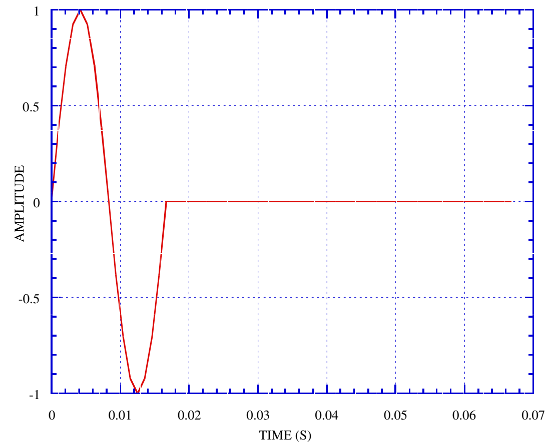

Figure 10: Time-Response to 60-Hz Unit Step Function

7.3 Adverse Effects on a Delta-Filter

A delta-filter should be tuned to a single frequency. Normally this is the rated network frequency: 50 or 60 Hz. Any change occurring after a fault on any frequency component other than the fundamental has an adverse effect on the delta-filter output.

A second important issue with conventional delta-filters is that the reference signal is constantly changing with time. Remember that we wish to subtract the waveform existing before the fault. In a situation where we have a succession of network changes that last longer that the filter delay, the reference signal no longer satisfies this requirement.

A last issue concerns the fact that some changes in a network topology cannot be handled by delta-filters. One example includes simultaneously energizing a line from both the local and remote terminals (such as a high-speed reclose). In this example, the delta-filter does not produce relevant superimposed quantities. Do not assume that the pre-event line currents have zero magnitude because the line did not "exist" electrically before the line breakers were closed.

7.4 Application of Delta-Filters to Phasors

Delta-filters can also be applied to phasors. The concept is illustrated in Figure 11. To accomplish this, you must have a time-invariant phasor or a phasor that remains still in the complex plane when no change occurs on the waveform. The delay implemented into the delta-filter need not be equal any longer than an integral number times the waveform period.

When no change is taking place on a network, the incremental or superimposed quantities are zero. We can take advantage of this property and implement a change detector using the delta-filter as shown in Figure 12. The magnitude of the incremental phasor is compared to a threshold INCR_TRH. When the change becomes greater than the threshold, a variable FREEZ indicating a change is set to 1. Due to the time delay drop-out, the variable remains asserted for a number of samples.

One of the shortcomings of the conventional delta-filter is its difficulty in coping with a succession of changes that last an interval of time longer than the delta-filter imbedded delay.

This situation is easily handled if the reference phasor, as shown in Figure 11, is maintained during the evolving events. To achieve this, we introduce the concept of the "double-windowed" delta-filter (patent pending) as represented in Figure 13. With this new principle, as soon as a change is detected, the value of the reference phasor is latched to a memory register. A second incremental quantity ΔV2 is then generated using the memorized phasor as its reference. The main property of this second incremental quantity is that its reference phasor is fixed. If a series of changes occur on the network, the reference is always the same when computing the incremental value.

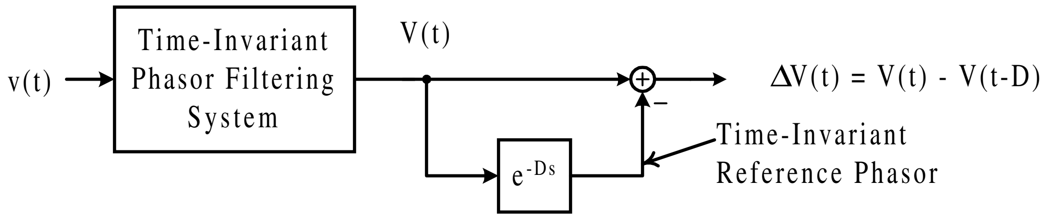

Figure 11. Concept of a Delta-Filter Applied to a Time-Invariant Phasor

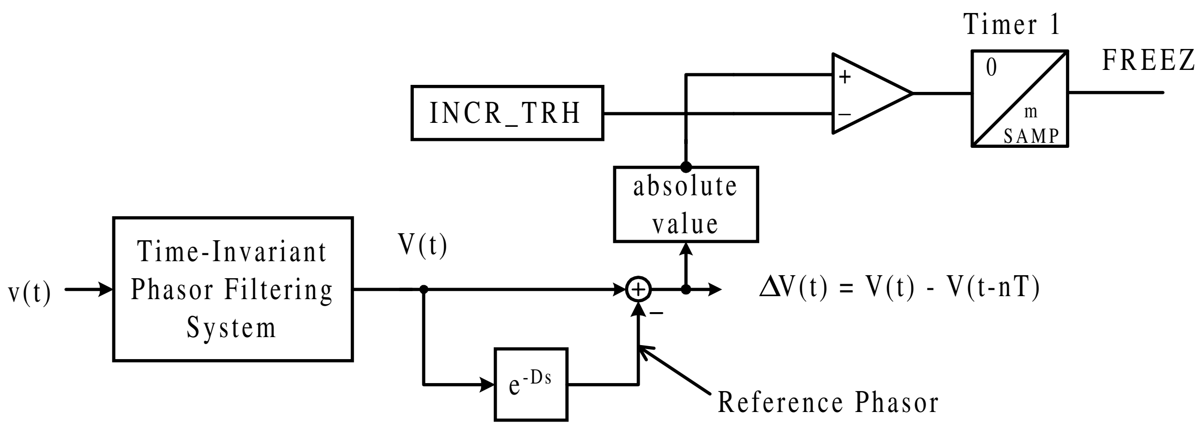

Figure 12. Concept of Delta-Filter Applied with a Change Detector

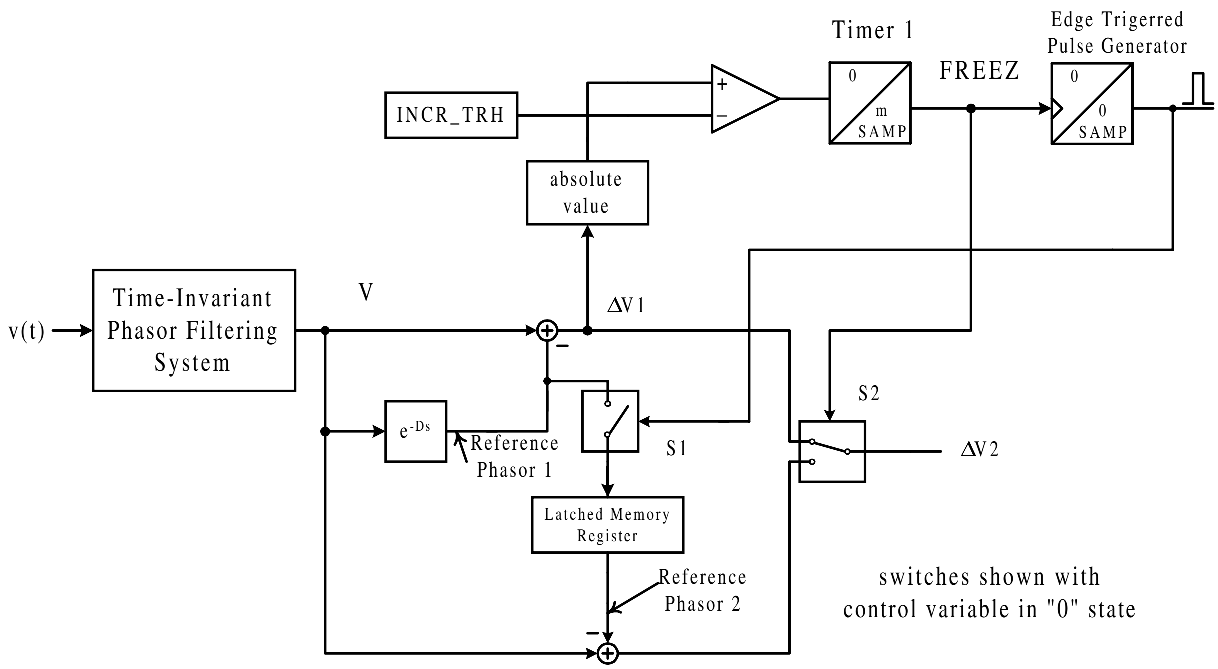

Figure 13. Concept of a "Double-Windowed" Delta-Filter

8. Implementation of Directional Elements in the Time and Frequency Domains

8.1 Implementation in the Time Domain

Using the scheme shown in Figure 14, we can implement a directional element which uses time domain superimposed quantities. The combination of the integrator and threshold detector establishes a phase angle comparison [9]. The phase angle comparison establishes the integrator output polarity: if the incremental voltage and the compensated incremental current waveforms are within ±90°, the integrator output is positive. The superimposed voltage and current are selected such that for a particular fault, the incremental impedance is equal to (–ZS1). The superimposed quantities are normally zero if no change occurs on the network. If a forward fault occurs, assume for the sake of simplicity, the incremental voltage at the delta-filter output is a sine wave as in:

$$\Delta vr(t) = \Delta v_R \bullet \sin(\omega t + \psi) \tag{8.1}$$

Using Equation 5.2 and accounting for any phase angle mismatch θ between the mimic and the source impedance, the incremental current after the mimic filter is provided by:

$$-\Delta irc(t) = \Delta i_R \bullet \sin(\omega t + \psi + \theta) \tag{8.2}$$

Integrating the product of the two incremental quantities results in the following equation:

$$COMP(t) = \frac{2}{T}\int_0^T \Delta v_R \bullet \sin(\omega t + \psi) \bullet \Delta i_R \bullet \sin(\omega t + \psi + \theta) \bullet dt \tag{8.3}$$

After an interval of time equal to one period, the integral has the value:

$$\text{COMP}(T) = \Delta v_R \bullet \Delta i_R \bullet \cos\ \theta \tag{8.4}$$

The integral output at the end of the integration period corresponds to the scalar product of Equation 5.4.

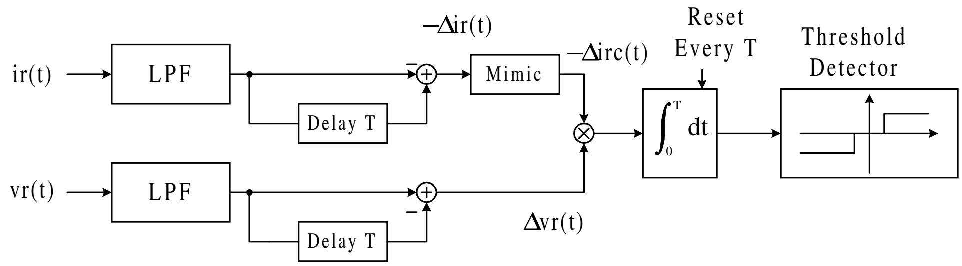

Figure 14: Time-Domain Generic Superimposed Quantities Directional Element

Figure 15 shows the integrator output COMP(t) for a forward fault with θ = 0° (perfect match between the mimic and the source impedance angles). Obviously the comparator output is positive from fault inception until time equals T. The basic issue regarding this type of comparator is the following: is the sign of the integrator output COMP(t) always the same as the sign of cos θ as time progresses from zero to T after fault inception?

To answer this question, let us look at the integrator output in Figure 16 for a reverse fault with ψ = 0° and an impedance mismatch θ varying from 90 to 180°. With an ideal phase comparator, the output should always be negative. The normalized (with unit incremental voltage and current) maximum positive value calculated by the comparator is 0.16. As noted in Reference [16], the integrator output should be compared to this same threshold before declaring a forward fault. Using the 0.16 threshold results in the following comparison:

$$\text{COMP}(t) > 0.16 \bullet \Delta v_R \bullet \Delta i_R \tag{8.5}$$

Figure 15 shows this 0.16 threshold. From Figure 15 notice the quick-response time: better than one-quarter-cycle, for a forward fault. There is, however, a shortcoming in this scheme. The threshold to which the integrator output has to be compared, incorporates the product of the incremental voltage and incremental current magnitudes. Thus, these two values must then be user-entered settings in a comprehensive scheme. The directional element sensitivity is also impacted: if a fault occurs, such that the subsequent changes in the voltage and the current are smaller than the entered settings, the relay does not make a directional declaration [16].

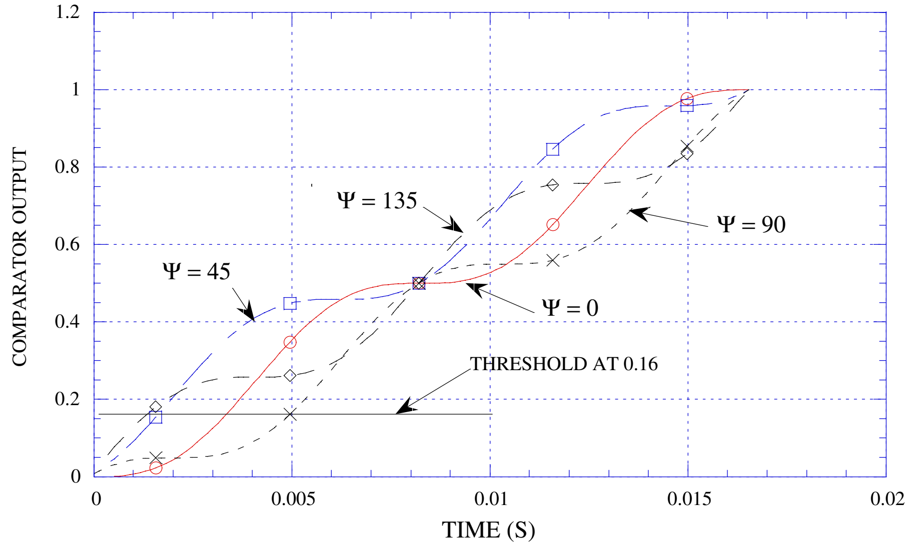

Figure 15: Comparator Output for a Forward Fault With θ = 0° and ψ Varying

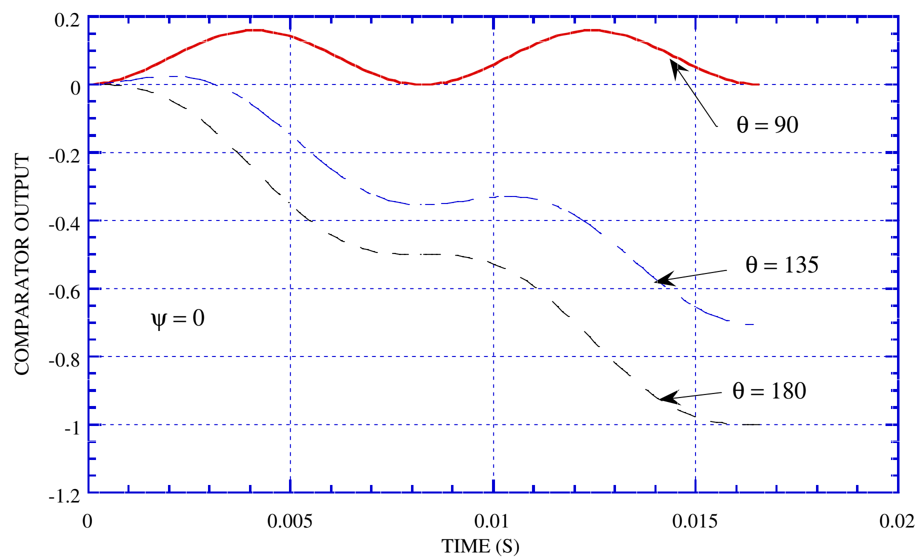

Figure 16: Comparator Output for a Forward Fault With ψ = 0° and θ = 90, 135, and 180°

8.2 Implementation in the Frequency Domain

The main advantage of implementing a superimposed directional element in the time domain is the speed achieved (theoretically, less than one-quarter-cycle). There are two drawbacks: practically no filtering and the anticipated voltage and current changes have to be defined as settings. These two shortcomings are overcome by implementing the directional element using frequency domain input quantities. The implementation of Equation 5.4 (referenced below as Equation 8.6), representing the basic principle of a directional element in the frequency domain (using phasors), is shown as a straightforward design in Figure 17.

$$\mathrm{real}(\Delta V_R \bullet \mathrm{conj}(\Delta I_R \bullet (\angle{-ZS1})) = \Delta v_R \bullet \Delta i_R \bullet \cos\theta \qquad (8.6)$$

The speed of the directional element now depends on the data-window of the selected filtering system. Fast direction assessment is still achieved. For example, with a one-half-cycle Fourier filtering system for phasor evaluation, the response time is less than one-half-cycle. Schemes using filtering are then superior to schemes implemented in the time-domain because there is no need to enter the anticipated changes as settings.

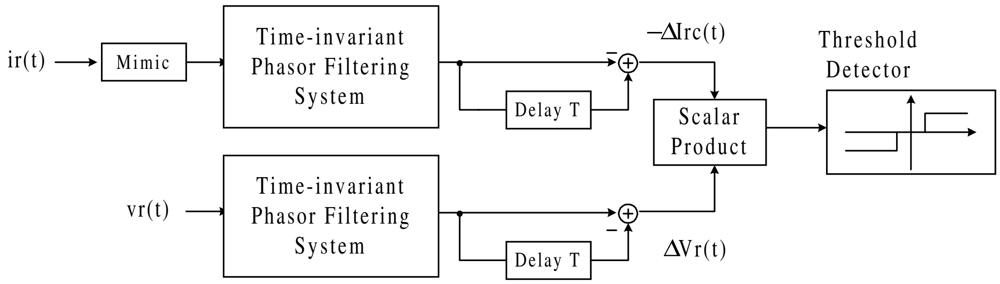

Figure 17: Frequency-Domain Generic Superimposed Quantities Directional Element

9. Implementation of Combined Phase-Selection and Directional Elements

Fast fault-type selection can be combined with directional assessment using the incremental impedances based on the differential (across two phases) superimposed voltages and currents. As shown in Table 2, for the single-line network (Figure 1), for any fault type, the incremental impedance is always equal to the negative of the source impedance behind the relay.

Table 2. Values of Differential Incremental Impedance

Fault Type	$\frac{\Delta V_{AB}}{\Delta I_{AB}}$	$\frac{\Delta V_{BC}}{\Delta I_{BC}}$	$\frac{\Delta V_{CA}}{\Delta I_{CA}}$
A-G	-ZS1	0 / 0	-ZS1
B-G	-ZS1	-ZS1	0 / 0
C-G	0 / 0	-ZS1	-ZS1
A-B, A-B-G	-ZS1	-ZS1	-ZS1
B-C, B-C-G	-ZS1	-ZS1	-ZS1
C-A, C-A-G	-ZS1	-ZS1	-ZS1
A-B-C	-ZS1	-ZS1	-ZS1

More useful and revealing information is obtained when the three incremental scalar products Δtab, Δtbc, and Δtca, corresponding to Equation 5.3 are performed. We define Δtab as:

$$\Delta tab = \mathrm{real}[\Delta V_{AB} \bullet \mathrm{conj}(\Delta I_{AB} \bullet (\angle -ZS1))] \quad (9.1)$$

In a practical application, the relay could assume that the local source impedance angle equals the angle of the positive-sequence line impedance. As described earlier, this can be done without changing the nature of the final results.

$$\angle ZS1 = \angle ZL1 \quad (9.2)$$

With the incremental compensated current defined as:

$$\Delta I_{ABc} = \Delta I_{AB} \bullet (1 \angle ZL1) \qquad (9.3)$$

where the current angular advance is provided by the mimic filter, we can now define the incremental scalar products as:

$$\Delta tab = \text{real}\,[\Delta V_{AB} \bullet \text{conj}\,(-\Delta I_{ABc})] \qquad (9.4)$$

$$\Delta tbc = \text{real}\,[\Delta V_{BC} \bullet \text{conj}\,(-\Delta I_{BCc})] \qquad (9.5)$$

$$\Delta tca = \text{real}\,[\Delta V_{CA} \bullet \text{conj}\,(-\Delta I_{CAc})] \qquad (9.6)$$

The relative values of the three incremental scalar products are shown in Table 3 [17] for conventional shunt faults. As an example, for an A-phase-to-ground fault, Δtab and Δtca are equal to some positive value and Δtbc equals zero. An A-phase-to-ground fault could unequivocally be inferred from the logic shown in Figure 18. In this diagram, CSTA is a constant number entered as a factory or user setting. To detect a reverse single-phase-to-ground fault, Δtab and Δtca must both be negative. In the case of a forward three-phase fault, all three scalar products are nearly equal and positive. The same logic applies to the other faults.

Table 3: Relation Between the Scalar Products

Fault Type	Δtab	Δtbc	Δtca
A-G	Δtab	0	Δtab
B-G	Δtab	Δtab	0
C-G	0	Δtbc	Δtbc
A-B, A-B-G	Δtab	0.25 • Δtab	0.25 • Δtab
B-C, B-C-G	0.25 • Δtbc	Δtbc	0.25 • Δtbc
C-A, C-A-G	0.25 • Δtca	0.25 • Δtca	Δtca
A-B-C	Δtab	Δtab	Δtab

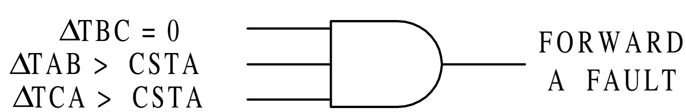

Figure 18: Logic to Establish a Forward Phase A-to-Ground Fault

10. Summary

Important points presented in this paper include the following:

1. Superimposed or incremental quantities belong to the pure-fault network as defined by the theory of linear circuits and superposition principle.

2. Sequence networks are useful in representing the pure-fault network if we subtract the load from positive-sequence current.
3. Phase comparators implemented in the time domain produce ultra-high-speed directional declarations. The user must enter the minimum anticipated delta-quantities.
4. Most shortcomings of high-speed directional elements using incremental quantities stem from the imbedded delta-filters limitations.
5. We can overcome these shortcomings using sub-cycle filtering system data-windows to derive the superimposed quantity phasors.
6. We showed how to combine rapid fault-type selection and directional declaration.

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