Tutorial: From the Steinmetz Model to the Protection of High Inertia Drives

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Published in
SEL *Journal of Reliable Power*, Volume 3, Number 1, March 2012

Previously presented at the
61st Annual Georgia Tech Protective Relaying Conference, May 2007,
60th Annual Conference for Protective Relay Engineers, March 2007,
and 6th Annual Clemson University Power Systems Conference, March 2007

Originally presented at the
33rd Annual Western Protective Relay Conference, October 2006

Tutorial: From the Steinmetz Model to the Protection of High Inertia Drives

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***Abstract*—The thermal limitations of induction motors are specified by thermal limit curves that are plots of the limiting temperature of the rotor and stator in units of I^2t. This paper discusses the thermal protection provided by rotor and stator thermal models defined by the thermal limit curves and supporting motor data. The thermal model is the time-discrete form of the differential equation for temperature rise due to current and is derived from fundamental principles, as shown in the Annex. In the rotor model, voltage and current are used to derive the slip-dependent I^2r watts that permit the safe starting of high-inertia drive motors.**

I. Introduction

Protection engineers are quite familiar with the coordination of inverse-time overcurrent characteristics to provide fault protection. Induction motors are thermally limited and also require thermal protection. This paper is a tutorial on thermal models used to calculate and monitor motor temperature to prevent overheating during starting and running conditions. The thermal models are the time-discrete form of the differential equation for temperature rise caused by current in a conductor. The models are derived from fundamental principles and rely on parameters defined by motor data. The model can be visualized as an electric analog circuit, and the temperature can be expressed in units of I^2t. Manufacturers specify the thermal limits using thermal limit curves that are I^2t plots of the limiting temperature. The curves for a 2250-horsepower, 3600-rotations-per-minute motor are shown in Fig. 1. The plots represent two initial conditions: the machine initially at ambient temperature and the machine initially at operating temperature. The thermal limit curves show only two of the possible conditions of a first-order thermal process where a balance of heat storage and heat loss determines temperature.

In the normal load case, the starting time is shorter than the locked rotor time. This allows the trip time of an inverse-time overcurrent relay to be set long enough to let the motor start yet short enough to prevent the current from exceeding the locked rotor time. However, increasing the moment of inertia increases the starting time, and the current encroaches on the locked rotor limit, as shown in Fig. 1.

This is the classic high-inertia case where the motor appears to overheat and the relay cannot be set to avoid a trip. This paper shows how voltage and current in the Steinmetz model are used to calculate the rotor resistance as a function of slip to determine the true heating during a high inertia start.

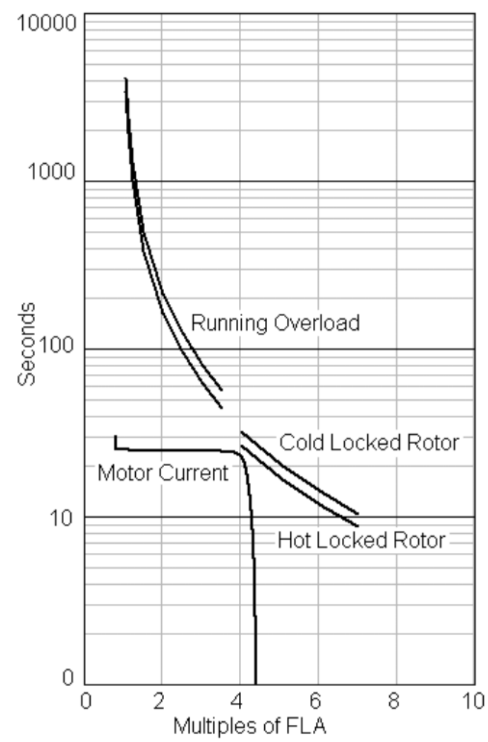

Fig. 1. 2250-Horsepower, 3600-Rotations-Per-Minute Motor Thermal Limit Curves

II. The Equivalent Circuit of the Induction Motor

The sources of motor heating are the watts loss in the resistance of the rotor and stator winding. The resistances are shown in the Steinmetz model of the motor in Fig. 2. R_S is the stator winding resistance. R_r is the slip-dependent rotor resistance that decreases from a high locked-rotor value to a low running value at rated speed.

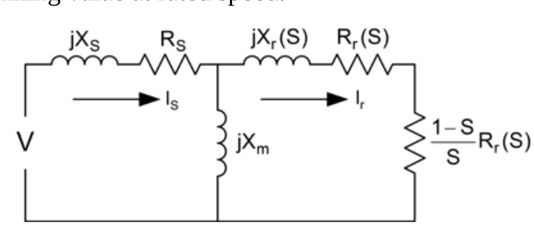

Fig. 2. Steinmetz Motor Equivalent Circuit

The positive- and negative-sequence rotor resistances are given by the linear functions of slip S:

$$R_1 = (R_M - R_N)S + R_N \tag{1}$$

$$R_2 = (R_M - R_N)(2 - S) + R_N \tag{2}$$

where: R_M is the rotor resistance at locked rotor
R_N is the rotor resistance at rated speed

R_M and R_N are known quantities defined by locked rotor current (I_L), locked rotor torque (LRQ), synchronous speed (ω_{syn}), and rated speed (ω_{rated}), as follows.

In the Steinmetz model shown in Fig. 2, the I^2r watts loss in the rotor resistor, $[(1\text{-}S)/S]R_r(S)$, is the mechanical power. Power P_M divided by speed $\omega = 1–S$ equals torque Q_M.

Therefore:

$$Q_M = \frac{P_M}{\omega} = \frac{P_M}{1-S} = I^2 \frac{1-S}{S} R_r \frac{1}{1-S} = \frac{I^2 R_r}{S} \tag{3}$$

Solving for R_r in terms of torque, current, and slip gives:

$$R_r = \frac{Q_M}{I^2} S \tag{4}$$

For locked rotor S = 1, Q_M = LRQ:

$$R_r = R_M = \frac{LRQ}{I_L^2} \tag{5}$$

The slip S at rated load is S_N, current I = 1 pu, and torque Q_M = 1 pu:

$$R_N = S_N \tag{6}$$

$$R_N = \frac{\omega_{syn} - \omega_{rated}}{\omega_{syn}} \tag{7}$$

$$R_M = \frac{LRQ}{I_L^2} \tag{8}$$

The rotor resistance for any value of slip can be calculated using values taken from the plot of current and torque shown in Fig. 3.

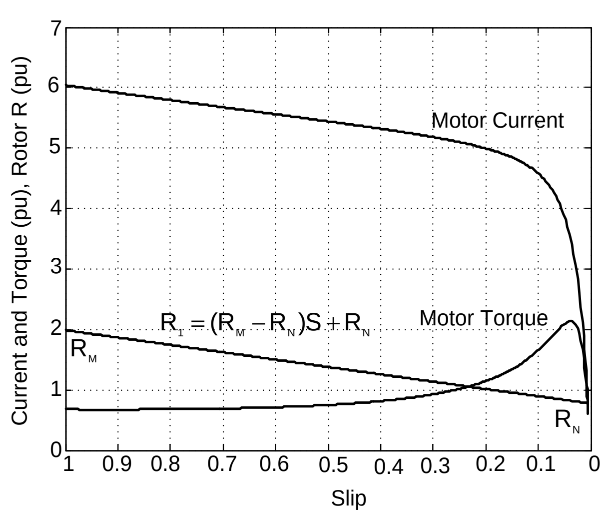

Fig. 3. Motor Current, Torque, and Rotor R Plotted Versus Slip

III. Rotor Thermal Model

Fig. 3 shows the excessively high current drawn until the peak torque drives the motor to full speed. A starting current of six times rated current and a locked rotor resistance R_M of three times R_N causes the I^2t heating of 6^2 x 3 = 108 times the heating of normal rated load current. Consequently, the extreme temperature caused by the high starting current must be tolerated for a limited time to allow the motor to start.

The safe starting time is indicated by the locked rotor curves shown in Fig. 1. The cold locked rotor characteristic specifies the time it takes the starting current to heat the rotor to the limiting temperature with the motor initially at ambient. The hot locked rotor characteristic specifies the time for starting current to heat the rotor to the limiting temperature with the motor initially at operating temperature. The limiting temperature in units of I^2t is:

$$U_L = I_L^2 T_A \tag{9}$$

where: U_L is rotor temperature limit
I_L is locked rotor current in per unit (pu) of FLA
T_A is safe stall time from ambient

Since both the hot and cold characteristics represent the same limiting temperature, the operating temperature can be expressed in terms of the limiting temperature as follows:

$$U_L = I_L^2 T_O + U_O \tag{10}$$

$$U_O = I_L^2 (T_A - T_O) \tag{11}$$

where: U_O is the operating temperature in I^2t
T_O is the safe stall time from operating temperature

Fig. 4 shows the first-order thermal model that incorporates the I^2t properties of the rotor thermal limit curves, as well as the effect of the slip-dependent positive- and negative-sequence rotor resistance on the input watts. The I^2t value of the operating temperature is used as the thermal resistance to ensure that one per unit input produces the operating temperature.

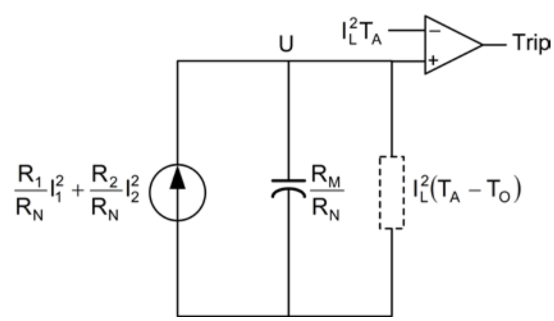

Fig. 4. Rotor Thermal Model

The following discrete form of the differential equation of the rotor thermal model is processed each processing interval to calculate the temperature U:

For I > 2.5:

$$U_n = \left(\frac{R_1}{R_N} I_1^2 + \frac{R_2}{R_N} I_2^2 \right) \frac{\Delta t}{C_{Th}} + U_{n-1} \tag{12}$$

For I <= 2.5

$$U_n = \left(\frac{R_1}{R_N} I_1^2 + \frac{R_2}{R_N} I_2^2 \right) \frac{\Delta t}{C_{Th}} + \left(1 - \frac{\Delta t}{R_{Th} C_{Th}} \right) \bullet U_{n-1} \qquad (13)$$

where: thermal capacitance $C_{Th} = R_M/R_N$
thermal resistance $R_{Th} = (I_L)^2(T_A - T_O)$
I_1 and I_2 are the positive- and negative-sequence currents, respectively.

Note that the thermal resistance is only considered when the current drops below 2.5 pu so that the calculation of temperature is adiabatic for starting current. At each sample, U_n is compared to the trip threshold and asserts the trip signal if the limiting temperature is exceeded.

Examples of the temperature U obtained from the rotor thermal model are shown in Fig. 5a and Fig. 5b. Note that the temperature is plotted in per unit of the limiting temperature U_L. Fig. 5a shows the locked rotor condition where R_r remains at its maximum value, and the I^2t temperature reaches the trip level in locked rotor time. Fig. 5b shows the successful start where R_r decreases, and the temperature reaches only 80 percent of the limiting temperature.

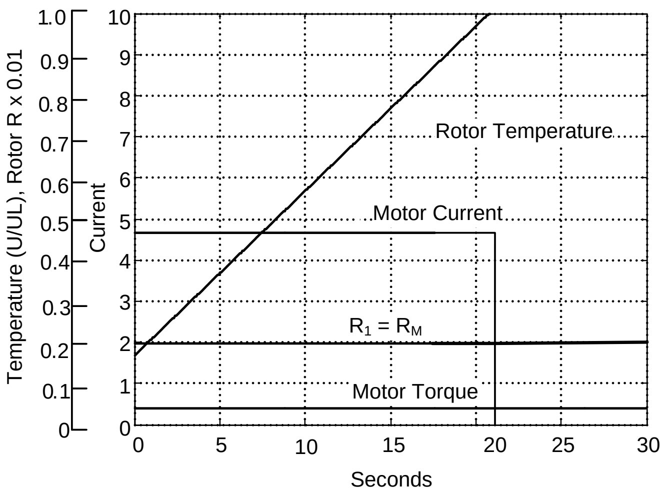

Fig. 5a. Locked Rotor Trip at Locked Rotor S = 1, $R_r = R_M$

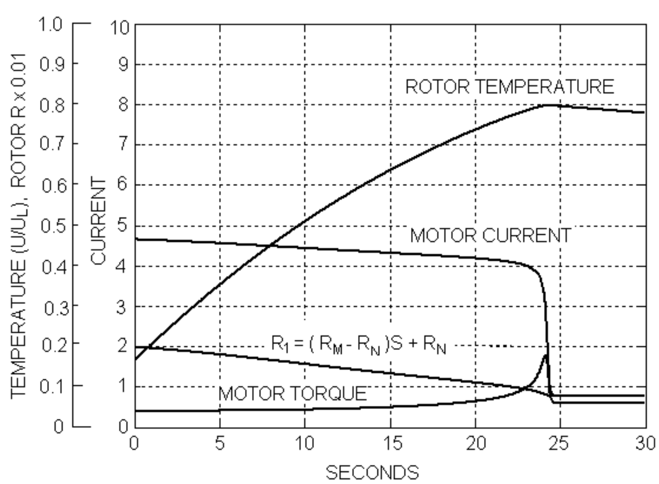

Fig. 5b. Motor Starting Current, Motor Torque R_r, and Temperature

IV. Calculating Slip

If the thermal model used a fixed rotor resistance R_M, it would produce an I^2t rise that overestimates the temperature during valid start. This is the cause of premature tripping when starting a high-inertia motor, as shown in Fig. 6. The figure shows the I^2t response of the relay reaching the trip threshold before the motor reaches running speed and the starting current subsides. The rotor reaches only 80 percent of the limiting temperature while the I^2t relay trips.

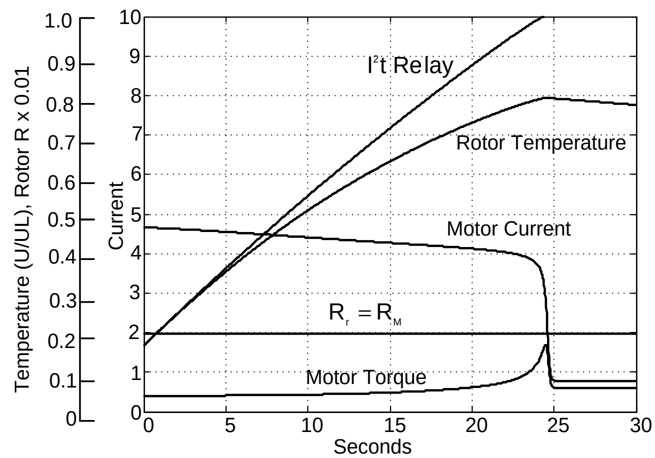

Fig. 6. Motor Starting Current With I^2t Temperature Response

To remedy this deficiency, the relay can use the measurement of voltage and current to calculate slip S. The slip can then be used to determine the slip-dependent rotor resistance. When motor voltage and current are monitored, the apparent positive-sequence impedance looking into the motor terminal is:

$$Z = R + jX = \frac{V_1}{I_1} \qquad (14)$$

From the Steinmetz equivalent circuit:

$$Z = R_S + jX_S + \frac{\left(\frac{R_r}{S} + jX_r \right) \bullet jX_m}{\frac{R_r}{S} + jX_r + jX_m} \qquad (15)$$

Expanding the equation:

$$Z = R_S + jX_S + \frac{\frac{R_r}{S} X_m^2 + j\left(X_m \left(\frac{R_r}{S} \right)^2 + X_r X_m \left(X_r + X_m \right) \right)}{\left(\frac{R_r}{S} \right)^2 + \left(X_r + X_m \right)^2}$$

The real part of Z is:

$$R = R_S + \frac{\frac{R_r}{S} \bullet X_m^2}{\left(\frac{R_r}{S} \right)^2 + \left(X_r + X_m \right)^2}$$

Dividing by $(X_m)^2$:

$$R = R_S + \frac{\frac{R_r}{S}}{\left(\frac{R_r}{S}\right)^2 \frac{1}{X_m^2} + \frac{(X_r + X_m)^2}{X_m^2}}$$

but $\left(\frac{R_r}{S}\right)^2 \frac{1}{X_m^2}$ is negligible.

Let:

$$A = \left(\frac{X_r + X_m}{X_m}\right)^2 \tag{16}$$

Using the real part of the motor impedance:

$$R = R_S + \frac{R_r}{A \cdot S} \tag{17}$$

substitute (1) for R_r in (17) and solve for slip S in terms of R_M, R_N, and the measured resistance R.

$$S = \frac{R_N}{A(R - R_S) - (R_M - R_N)} \tag{18}$$

The slip is then used in the positive- and negative-sequence resistance equations (1) and (2). The resistance of the rotor thermal model will then be slip-dependent and produce the slip-dependent temperature rise shown in Fig. 7.

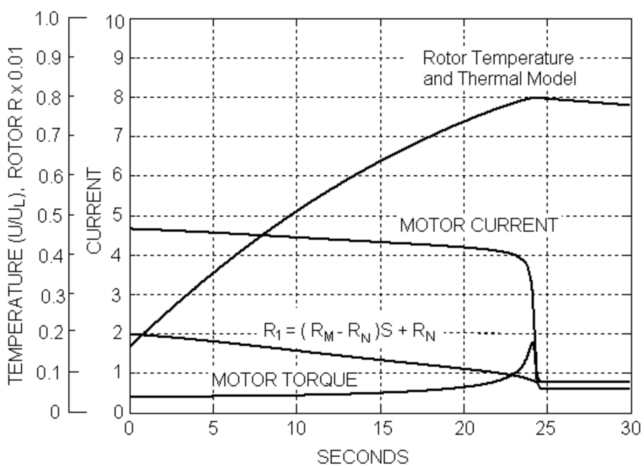

Fig. 7. Motor Starting Showing Thermal Model Emulating the Rotor Temperature

V. Overload Heating

The running overload curves in Fig. 1 show the thermal limit of the stator. The curves fit the time-current equation:

$$t = \tau \bullet \ln\left(\frac{I^2 - I_0^2}{I^2 - SF^2}\right) \tag{19}$$

where: τ is the stator thermal time constant
I is the stator current pu of FLA
I_0 is the initial current in pu of FLA
SF is the motor service factor

The overload curves in Fig. 1 are asymptotic to the current equal to the service factor that heats the stator to its temperature limit and is taken as the trip threshold. The stator thermal time constant can be determined by a heat run, where a load current is applied and the rise is measured at regular time intervals. The temperature will rise exponentially, and the thermal time constant will be the time it takes the temperature to reach 63.2 percent of its final value. In the case of the 2500-horsepower motor, the time constant τ is 5066 seconds.

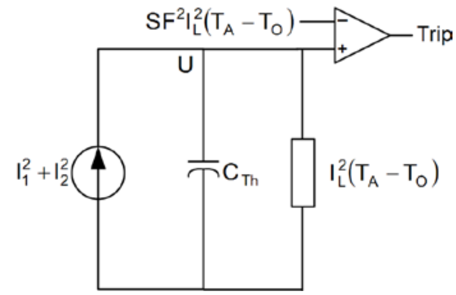

Fig. 8. Stator Thermal Model

The stator thermal model is shown in Fig. 8. $I_L^2(T_A - T_O)$ is the operating temperature in units of I^2t and is used as the thermal resistance R_{Th} because 1.0 pu current flowing in the thermal resistance produces rated operating temperature. Consequently, the trip threshold is SF^2 times the operating temperature. Therefore, the thermal capacitance C_{Th} is:

$$C_{Th} = \frac{\tau}{I_L^2(T_A - T_O)} \tag{20}$$

The following discrete form of the differential equation of the stator thermal model is processed each processing interval to calculate the temperature U:

$$U_n = \left(I_1^2 + I_2^2\right) \bullet \frac{\Delta t}{R_{Th}C_{Th}} + \left(1 - \frac{\Delta t}{R_{Th}C_{Th}}\right) \bullet U_{n-1} \tag{21}$$

VI. Stator Time Constant

The time-current equation in (19) gives the time it takes a constant current to raise the stator temperature to the trip level, starting from the temperature caused by the previous load current I_0. Note that the position of the overload curve is determined by the value of I_0 for which the manufacturer chooses to plot the curve.

The time constant τ is the key parameter of the stator thermal model. When not specified, a reasonable estimate can be made as follows. Assume the motor had a previous load of 0.95 pu current when the motor is started. Where both stator and rotor are heating, under a locked rotor condition, we would expect the rotor to trip before the stator. To guarantee this condition with locked rotor current, let the stator thermal model trip in the cold locked rotor time:

$$t = T_A = \tau \bullet \ln\left(\frac{I_L^2 - 0.95^2}{I_L^2 - SF^2}\right) \tag{22}$$

$$\tau = \frac{T_A}{\ln\left(\frac{I_L^2 - 0.95^2}{I_L^2 - SF^2}\right)} \quad (23)$$

For the 2250-horsepower motor, I_L = 5.9375, SF = 1.0, T_A = 14.4:

$$\tau = \frac{14.4}{\ln\left(\frac{5.9375^2 - 0.95^2}{5.9375^2 - 1^2}\right)} = 5066$$

VII. Settings of the Thermal Model

The settings are the thermal time constant τ and the service factor SF:

τ	= 5066 s
SF	= 1.0

For rotor model:

ω_{syn}	= 3600 rpm	Syn, Speed
ω_{rated}	= 3572 rpm	Rated Speed
I_L	= 5.975 pu	Locked Rotor Current
LRQ	= 0.7 pu	Locked Rotor Torque
T_A	= 14.4 s	Cold Rotor Limit
T_O	= 12.0 s	Hot Rotor Limit

The relay monitors voltage and current to determine the motor Z and calculates R_M and $R_{N.}$ The real part of Z is then used to derive slip and calculate the slip-dependent rotor positive- and negative-sequence resistance (see Section V).

$$R_N = \frac{\omega_{syn} - \omega_{rated}}{\omega_{rated}} = \frac{3600 - 3560}{3600} = .0111$$

$$R_M = \frac{LRQ}{I_L^2} = \frac{0.7}{5.5^2} = .0231$$

With these settings, the stator and thermal models take on the dynamic thermal properties of the 2250-horsepower motor.

The speed algorithm applied to the 2250-horsepower motor is as follows:

$$R = \text{real}\left(\frac{V1}{I1}\right) \quad (24)$$

where: V1 and I1 are the positive-sequence motor voltage and current measured at each processing interval.

$$A = 1.2 \quad (25)$$

R_M and the initial value of R measured during start are used to determine the stator resistance R_S which includes the source resistance:

$$R_S = R(1) - \frac{R_M}{A} \quad (26)$$

then

$$S = \frac{R_N}{(A(R - R_S) - (R_M - R_N))} \quad (27)$$

and

$$R_r = (R_M - R_N)S + R_N \quad (28)$$

The produced plots of R, S, and R_r are shown in Fig. 9, Fig. 10, and Fig. 11.

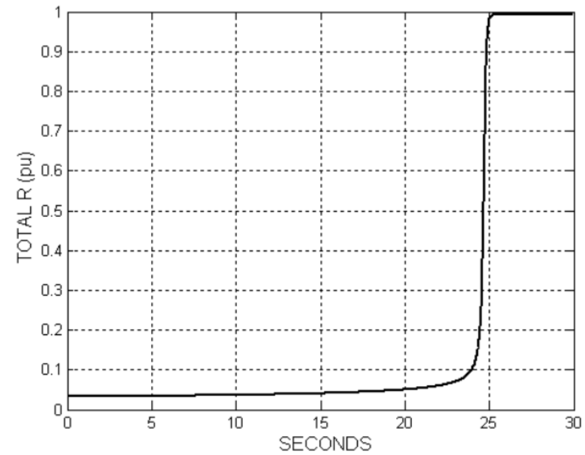

Fig. 9. Total Motor Resistance During Start

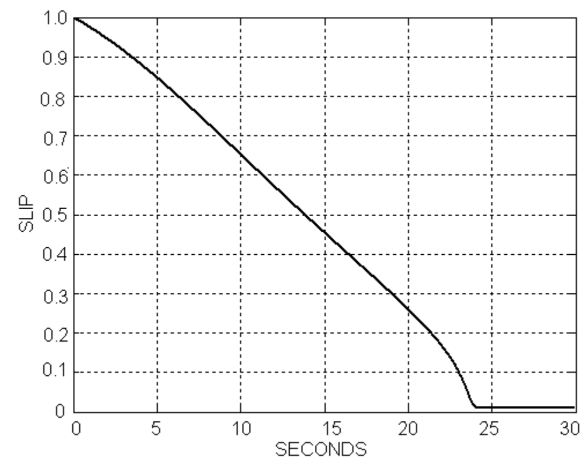

Fig. 10. Motor Slip During Start

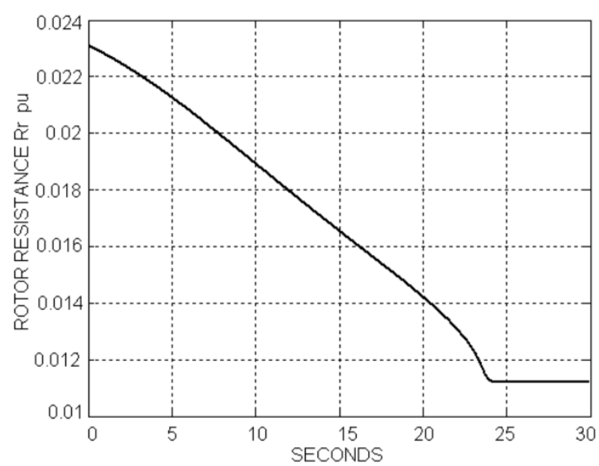

Fig. 11. Rotor Resistance During Start

VIII. 2250-Horsepower Motor Test Data

Fig. 12, Fig. 13, Fig. 14, and Fig. 15 show the results of a full-scale test of a 2250-horsepower compressor motor start.

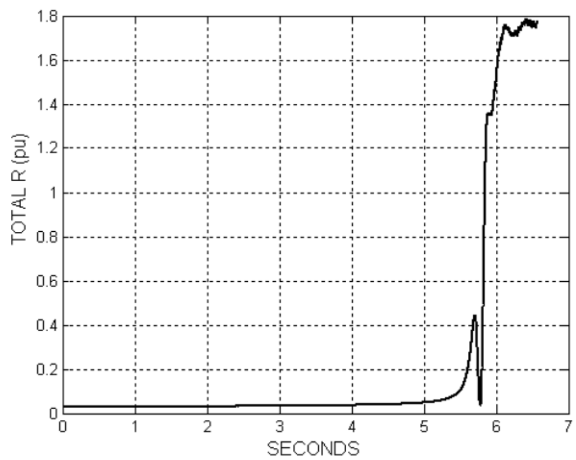

Fig. 12. 2250-Horsepower Motor Start Total R

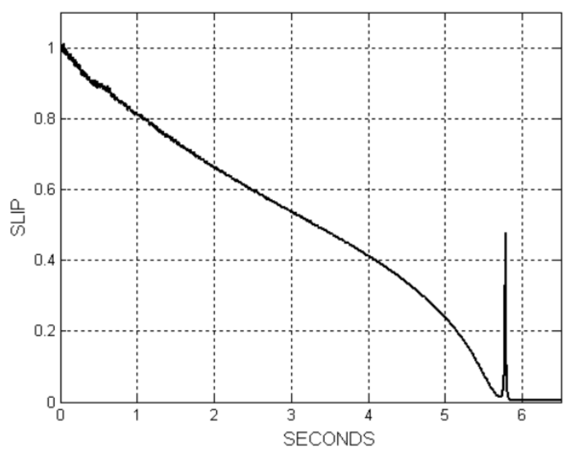

Fig. 13. 2250-Horsepower Motor Start Slip

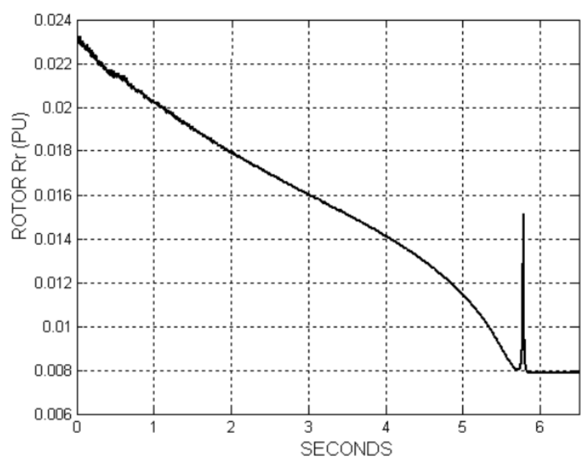

Fig. 14. 2250-Horsepower Motor Start Rotor Resistance R_r

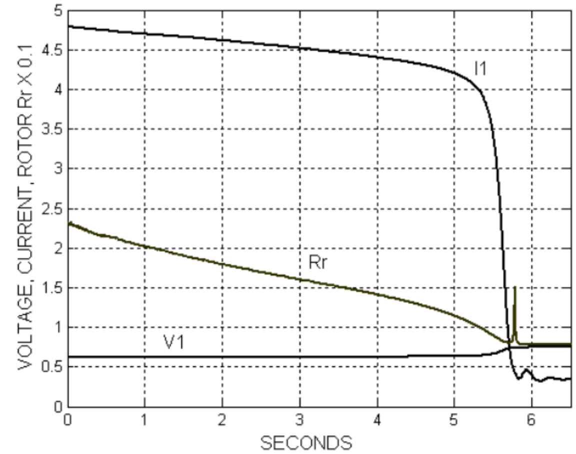

Fig. 15. 2250-Horsepower Motor Positive-Sequence Voltage V1, and Current I1 and Slip-Dependent Rotor Resistance

IX. Annex: The First Order Thermal Model

The first order thermal model is derived as follows:

$$\theta = \theta_w - \theta_A \tag{29}$$

where: θ is defined as the winding temperature rise θ_W above ambient temperature θ_A

The rate of increase of the temperature is given by the equation expressing the thermal equilibrium.

$$\text{Power Supplied} - \text{Losses} = C_s m \frac{d\theta_W}{dt} = C_s m \frac{d\theta}{dt} \tag{30}$$

In this equation, C_s is the specific heat of the winding, and *m* is the mass. The specific heat corresponds to the amount of energy needed to raise one kilogram of that material one degree centigrade. The losses or the quantity of heat transferred to the surrounding environment is expressed as:

$$\text{Losses} = \frac{\theta_W - \theta_A}{R} = \frac{\theta}{R} \tag{31}$$

where: R is the thermal resistance in °C/watt.

Equation (30) can be otherwise expressed as:

$$I^2 r - \frac{\theta}{R} = C_s m \frac{d\theta}{dt} \tag{32}$$

or

$$I^2 r \bullet R = \theta + C_s m \bullet R \frac{d\theta}{dt} \tag{33}$$

The mass *m* multiplied by the specific heat C_s is known as C, the thermal capacity of the system with units of joules/°C. It represents the amount of energy in joules required to raise the system temperature by one degree centigrade.

The product of the thermal resistance R and the thermal capacitance C has units of seconds and represents the thermal time constant:

$$\tau = R \bullet m \bullet C_s \tag{34}$$

The fundamental equation (33) can be expressed in a simpler form:

$$I^2 = C_s m \bullet R \bullet \left(\frac{1}{r \bullet R} \bullet \frac{d\theta}{dt} \right) + \frac{\theta}{r \bullet R} \tag{35}$$

$$\tau = C_s m \bullet R \tag{36}$$

let

$$U = \frac{\theta}{r \bullet R} \tag{37}$$

and

$$\frac{dU}{dt} = \frac{1}{r \bullet R} \bullet \frac{d\theta}{dt} \tag{38}$$

Therefore, the first order thermal model equation becomes the simple form:

$$I^2 = \tau \frac{dU}{dt} + U \tag{39}$$

The solution of the first order equation is:

$$U = I^2 \bullet \left(1 - e^{\frac{-t}{\tau}} \right) \tag{40}$$

With initial current I_0:

$$U = I^2 \bullet \left(1 - e^{\frac{-t}{\tau}} \right) + I_0^2 \bullet e^{\frac{-t}{\tau}} \tag{41}$$

When using (41) to calculate U over a small time increment Δt, the exponentials can be replaced with the first two terms of the infinite series as follows:

$$e^{\frac{-\Delta t}{\tau}} = \left(1 - \frac{\Delta t}{\tau} \right) \tag{42}$$

Substituting (42) in (41) gives:

$$U_n = I^2 \bullet \left(1 - \left(1 - \frac{\Delta t}{\tau} \right) \right) + U_{n-1} \bullet \left(1 - \frac{\Delta t}{\tau} \right) \tag{43}$$

This incremental form of the equation is ideal for use in the processor for the continuous real-time calculation of temperature:

$$U_n = \frac{I^2 \Delta t}{\tau} + \left(1 - \frac{\Delta t}{\tau} \right) \bullet U_{n-1} \tag{44}$$

where: U_n is the temperature expressed in units of I^2t at sample *n*
U_{n-1} is the temperature expressed in units of I^2t at the previous sample

Electrical engineers find it helpful to visualize the thermal model as an electrical analog circuit. The first order equation of the thermal model has the same form as the equation expressing the voltage rise in an electrical RC circuit, as shown in Fig. 16.

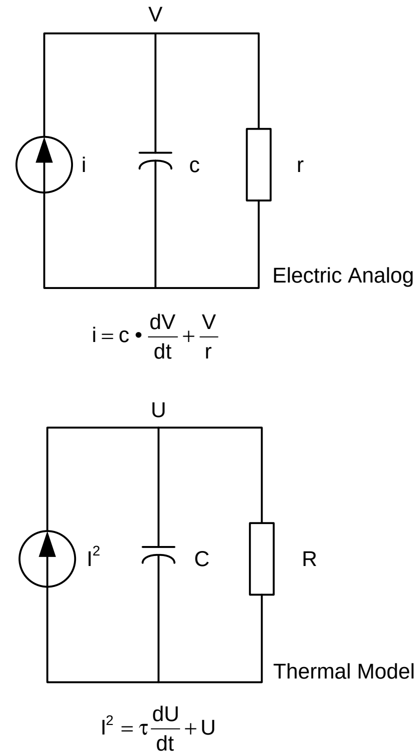

Fig. 16. The Electrical Analog Circuit of the Thermal Model

In the figure, the lowercase letters are used to identify the electrical parameters. In the circuit, the voltage is the analog of the temperature U, the constant current is numerically equal to the current squared. The thermal resistance R and thermal capacitance C are the direct analogs of the electrical resistance r and the electrical capacitance c.

X. Conclusions

1. Motor thermal limit curves are plots of the limiting temperature of the rotor and stator expressed in units of I^2t.
2. Stator and rotor thermal first order models are the differential equations for heat rise in a conductor that calculates temperature rise in real time.
3. Thermal limit curves and supporting motor data define the thermal models.
4. Voltage and current monitored by a motor relay are used to calculate slip and the slip-dependent rotor resistance so that the protection of high-inertia drive motors is inherent.

XI. References

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XII. Biography

Stanley (Stan) Zocholl has a B.S. and M.S. in Electrical Engineering from Drexel University. He is an IEEE Life Fellow and a member of the Power Engineering Society and the Industrial Application Society. He is also a member of the Power System Relaying Committee. He joined Schweitzer Engineering Laboratories, Inc. in 1991 in the position of Distinguished Engineer. He was with ABB Power T&D Company Allentown (formerly ITE, Gould, BBC) since 1947 where he held various engineering positions including Director of Protection Technology.

His biography appears in Who's Who in America. He holds over a dozen patents associated with power system protection using solid state and microprocessor technology and is the author of numerous IEEE and Protective Relay Conference papers. He received the Power System Relaying Committee's Distinguished Service Award in 1991. He was the Chairman of PSRC WG J2 that completed the AC Motor Protection Tutorial. He is the author of the books *AC Motor Protection*, second edition, ISBN 0-9725026-1-0 and *Analyzing and Applying Current Transformers*, ISBN 0-9725026-2-9.

Previously presented at the 2007 Texas A&M
Conference for Protective Relay Engineers.

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