# Synchrophasor-Based Online Modal Analysis to Mitigate Power System Interarea Oscillation

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***Abstract*—Power system interarea oscillations constrain the amount of power that can be transferred through some parts of interconnected power grids. Interarea oscillations without proper remedial action can result in power system separation or a major blackout. This paper reports on a new technology that can automatically mitigate unstable power system oscillations. The system consists of real-time modal analysis based on power system synchrophasor measurements, interarea oscillation identification logic, and remedial action logic. The oscillation identification logic indentifies the interarea oscillation modes across the power system and detects any unstable oscillation mode. The identification logic and control logic are implemented in standard IEC 61131 programming language. With minimal configuration, the system can provide automatic early warning of unwanted power system oscillations and trigger appropriate remedial actions. Real-time digital simulation (with hardware in the loop) test results demonstrate the feasibility of this new technology.**

## I. Introduction

Power system interarea oscillation is a major concern to large utility power companies. It imposes constraints on the amount of power that can be transferred through some parts of the interconnected power grids and, therefore, can negatively impact the economical operation of the power system. Without proper remedial actions, interarea oscillations can result in power system separation or major blackouts.

The traditional approach to preventing unstable interarea oscillations involves analysis of power system dynamic simulation results at the planning stage. There are two major categories of modal analysis methods. One category is based on the eigenvalue analysis of the state space model of the power system dynamic model. The state space model normally results from linearizing the system around the operating point. If any eigenvalue of the transfer matrix has a positive real part, the system exhibits an increasing oscillation problem and is unstable around the operating point [1]. The second category involves applying signal processing algorithms, such as Prony analysis [2] and spectral analysis, on such simulated power system variables as reactive power, voltage magnitude, and phase angle difference after a power system disturbance. Generally, these signal processing algorithms decompose the measurement of variables over a period of time into multiple oscillation modes characterized by amplitude, frequency, and damping ratio. Presence of a mode with negative damping ratio indicates that the system has an increasing oscillation problem. The second approach is in common use for post-disturbance analysis to identify oscillation modes. Accuracy of the power system dynamic models and the number of contingencies and operation conditions available for study limit these traditional approaches.

We now can use advanced computing and signal processing technology to implement synchrophasor-based power system modal analysis in real time. The synchrophasor vector processor (SVP) implements the modified Prony analysis as a built-in modal analysis (MA) function block. The SVP internally time-aligns synchrophasor data from different phasor measurement and control units (PMCUs) and makes this information available to different applications. The user can select a different synchrophasor measurement as the input to each modal analysis function block and implement decision and control logic in real time. In this paper, we briefly present the modified Prony algorithm and a system integrity protection system (SIPS) that automatically detects unstable power system interarea oscillation and triggers predefined remedial action schemes to mitigate unstable oscillations.

## II. Modified Prony Analysis

Prony analysis is a technique for modeling evenly sampled data as a linear combination of exponentials. For an array of data samples x[1], …, x[N], the Prony method estimates x[n] according to (1).

$$\hat{x}[n] = \sum_{k=1}^{M} A_k e^{\alpha_k (k-1)T} \cos(2\pi f_k (k-1)T + \theta_k) \quad (1)$$

For $1 \le n \le N$, where $T$ is the sample interval in seconds, $A_k$ is the amplitude of the exponential, $\alpha_k$ is the damping constant in seconds$^{-1}$, $f_k$ is the frequency in Hz, $\theta_k$ is the initial phase in radians, and $M$ is the number of exponential modes. Original Prony analysis fits as many exponential modes as half of the number of data samples. For modified Prony analysis, the number of data samples $N$ usually exceeds twice the number of estimated modes [1]. We can express (1) also by (2).

$$\hat{x}[n] = \sum_{k=1}^{M} h_x z_k^{n-1} + h_k^* (z_k^*)^{n-1} \quad (2)$$

where * stands for the conjugate operation, and $h_k$ and $z_k$ are defined as follows:

$$h_k = \frac{A_k}{2} e^{j\theta_k} \quad (3)$$

$$z_k = e^{(\alpha_k + j2\pi f_k)T} \quad (4)$$

The polynomial equation with roots $z_k$ and $z_k^*$ has the form of (5).

$$\prod_{k=1}^{M}(z-z_k)(z-z_k^*) = \sum_{k=0}^{m} a[k]z^{m-k} = 0, \quad (5)$$

where m = 2M , *a[0]* is usually normalized to 1, and the other *a[k]*s are the solution of linear equations such as (6).

$$\begin{bmatrix} x[m-1] & x[m-2] & \cdots & x[0] \\ x[m-0] & x[m-1] & \cdots & x[1] \\ \vdots & \vdots & \ddots & \vdots \\ x[N-2] & x[N-3] & \cdots & x[N-m-1] \end{bmatrix} \times \begin{bmatrix} a[1] \\ a[2] \\ \vdots \\ a[m] \end{bmatrix} = \begin{bmatrix} x[m] \\ x[m+1] \\ \vdots \\ x[N-1] \end{bmatrix} \quad (6)$$

The first matrix in (6) is called signal matrix *S*, *a[k]* is coefficient vector *A*, and the right side of (6) is signal vector *X*.

Modified Prony analysis involves four major calculation steps.

### A. Determine Polynomial Equation Coefficients

Equation (6) is an overdetermined linear equation. For an ideal signal with *K* (*K* < *M*) real exponentials, the signal matrix will have rank *2K*. If the signal has additive noise, the signal matrix will have the full rank of *2M*. To minimize the impact of noise on the numerical accuracy of the modified Prony analysis, we use singular value decomposition (SVD) based on the least square method to solve (6). We replace all eigenvalues smaller than the eigenvalue cutoff threshold with zeros and treat the corresponding eigenvectors as noise vectors. We can write the eigendecomposition of the signal matrix as (7).

$$S = \sum_{i=1}^{L} \lambda_i u_i v_i^* \quad (7)$$

where eigenvalues are in decreasing value $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_L \geq \lambda_{cutoff}$ , and $u_i$ *and* $v_i$ are left- and right-singular vectors for the corresponding singular value. Then we can solve the coefficient vector *A* according to (8).

$$A = \sum_{i=1}^{L} \frac{1}{\lambda_i} v_i u_i^* X \quad (8)$$

### B. Solve Polynomial Equation

Existing algorithms for univariate polynomial equations are either iterative algorithms or theoretically complete algorithms. To solve (5) with typical order of tens, iterative algorithms perform well [4]. Among these iterative algorithms, the companion (Frobenius) matrix approach has solid theoretical ground. Many mathematical solvers, such as Matlab and Mathematica, have adopted this method for calculating the roots of high-order polynomial equations. The eigenvalues $\lambda_i$ ($1 \leq i \leq m$) of the companion matrix (9), built from (2), are the numerical solutions of the roots from (5). $\lambda_i$ is either a real number or a member of conjugate complex numbers.

$$\begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ & \vdots & & \ddots & \\ -a[m] & -a[m-1] & -a[m-2] & \cdots & -a[1] \end{bmatrix} \quad (9)$$

We use (10) and (11) to determine the damping factor, $\alpha_i$, and frequency, $f_i$, of each mode.

$$\alpha_i = \frac{Re(\log \lambda_i)}{T} \quad (10)$$

$$f_i = \frac{Im(\log \lambda_i)}{2\pi T} \quad (11)$$

### C. Determine Mode Amplitude and Initial Phase

Use the roots of (5) to construct the first matrix element of (12), which we can then use to calculate the residual vector $R_i$s. Then we use (12) and (13) to calculate the amplitude $A_i$ and initial phase of each calculated mode.

$$\begin{bmatrix} \lambda_1^0 & \lambda_1^0 & \cdots & \lambda_1^0 \\ \lambda_1^0 & \lambda_1^0 & \cdots & \lambda_1^0 \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_1^0 & \lambda_1^0 & \cdots & \lambda_1^0 \end{bmatrix} \times \begin{bmatrix} R_1 \\ R_2 \\ \vdots \\ R_m \end{bmatrix} = \begin{bmatrix} x[0] \\ x[1] \\ \vdots \\ x[N-1] \end{bmatrix} \quad (12)$$

$$A_i = |R_i| \quad (13)$$

$$\theta_i = \tan^{-1} \frac{Im(R_i)}{Re(R_i)} \quad (14)$$

All the data samples are real numbers, so only the modes with frequency $f_i \geq 0$ are valid. Accordingly, the true amplitudes of the valid modes with $f_i > 0$ are twice as large as the amplitudes (12) calculates.

### D. Determine the Quality of the Modified Prony Analysis Fit

The modified Prony analysis is a linear approximation-based, curve-fitting technique. It uses the linear combination of multiple exponential oscillation modes to approximate the original signal. We can use (1) to reconstruct the estimated signal $\hat{x}[n]$. The signal-to-noise ratio (SNR) calculated in (15) quantifies the quality of the curve fitting [2].

$$SNR = 10log_{10}\left(\frac{\sum_{n=0}^{N} x[n]^2}{\sum_{n=0}^{N}(x[n]-\hat{x}[n])^2}\right) \quad (15)$$

Modified Prony analysis produces a low SNR value if the data sample array contains nonlinear transitions. In power systems, discrete switching events, such as line tripping, can cause nonlinear transitions. The SNR value normally improves as a switching event leaves the observation window of modal analysis, and the power system settles into pure oscillation mode. A high SNR value (greater than 80 dB, for example) indicates that the analysis result is a good approximation of the original signal.

## III. Interarea Oscillation Mode Identification

Power system disturbances, such as line tripping and drop of generation, cause local and interarea oscillations. The oscillations are visible power system measurements. Usually, local oscillation modes range in frequency from 0.7 to 2.0 Hz. Interarea oscillation, which refers generally to a group of generators in one area that swing against a group of generators in another area, normally ranges in frequency from 0.1 to 0.8 Hz [5].

The modal analysis function block in SVP reports results that include an array of modes and the signal-to-noise ratio.

Each mode is a data structure that includes amplitude, frequency $f$ in Hz, damping constant α, damping ratio ζ, and initial phase angle $\theta$ in degrees. We can use (17) to calculate the damping ratio from frequency and damping constant.

$$\zeta = \frac{-\alpha}{\sqrt{\alpha^2+(2\pi f)^2}} \tag{17}$$

A negative damping ratio indicates that the corresponding mode is an increasing oscillation mode. The array of calculated oscillation modes includes both local oscillation modes and interarea oscillation modes. To identify interarea oscillation modes from the array of modes, additional logic must process the modal analysis results based on measurement from different areas before control actions can occur. Fig. 1 illustrates the decision-making process based on modal analysis results.

Modified Prony analysis involves numerical approximation, so the calculated mode frequencies from different areas can vary for a common interarea oscillation mode. Therefore, the process of identifying the common oscillation modes normally uses a frequency deviation threshold. We can identify all oscillation modes with frequency variation from their mean value less than the frequency deviation threshold as one common mode.

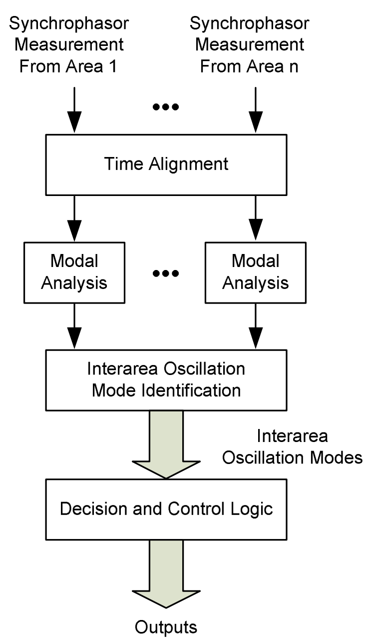


Fig. 1. Remedial action based on modal analysis results

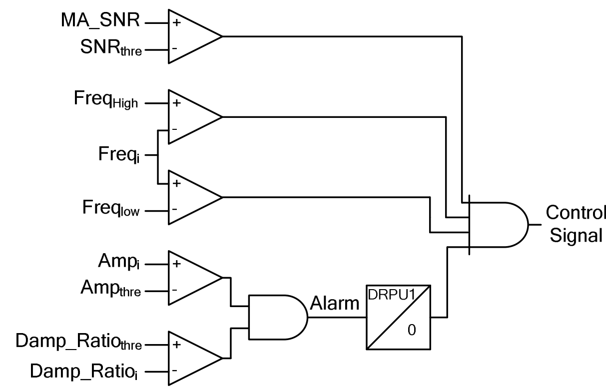


Fig. 2. Oscillation mode-based decision logic

We can then feed the parameters of the common interarea oscillation mode we identify into customized decision logic to initiate alarm output signals or control output signals. Fig. 2 illustrates an example of decision and control logic. The input $SNR_{thre}$ determines the minimum modal analysis signal-to-noise ratio to enable the output. The $Freq_{high}$ and $Freq_{low}$ thresholds define the frequency band of the interarea oscillation mode in which we are interested. If the mode amplitude is greater than $Amp_{thre}$ and the damping ratio exceeds $Damp\_Ratio_{thre}$, the alarm asserts. If the alarm condition persists longer than *DRPU1* seconds, then the control output becomes true.

All the modal analysis results together with the time-aligned synchrophasor measurement are directly available to the SVP's run-time engine. Users can use IEC 61131 programming language to develop more sophisticated identification and control logic tailored for their system characteristics.

## IV. Performance of Modal Analysis Detecting Interarea Oscillation Mode

Here we use a two-area power system model, as in Fig. 3, to test system performance. Past literature has used this power system model with small variations to illustrate the interarea oscillation problem [1] [6]. The system parameters we use here are almost identical to [6], except that we modeled the load as a constant load ($V > 0.80$ pu) instead of as a constant impedance to obtain more realistic results. The power system consists of two areas that are connected through two parallel transmission lines. Each area has two synchronous generators equipped with a governor and an excitation controller. Two relays with synchrophasor measurement and control capabilities monitor the voltages and currents of the intertie. The analytic result shows that three dominant oscillation modes exist in the system. Each area has a local oscillation mode, and there is one interarea oscillation mode [1]. Appendix A lists the generator outputs and loads in the base case.

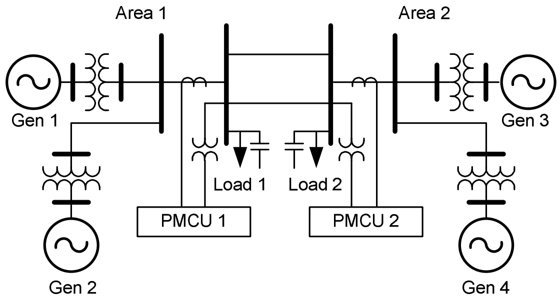


Fig. 3. Two-area power system model with an interarea oscillation problem

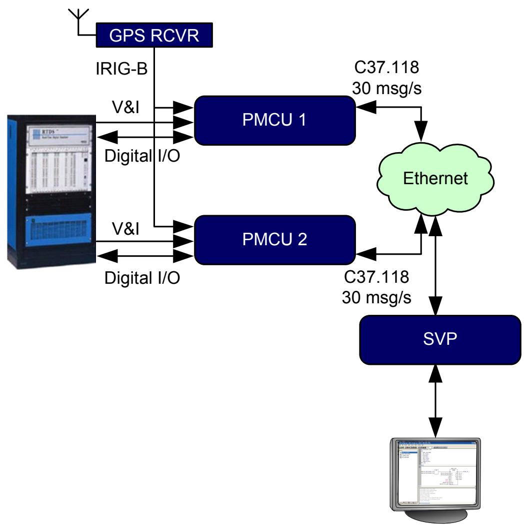


Fig. 4. Power system model in real-time digital simulator with synchrophasor data acquisition and SVP

We used a real-time digital simulator (RTDS®) to simulate power system dynamics. Fig. 4 shows the system to evaluate the scheme. Two relays measure the currents and voltages in real time at both sides of the intertie. RTDS and SVP exchange the device status and control signal through the relay to make the setup a real-time, closed-loop-controlled power system. The relays send synchrophasor measurements (voltages, currents, and device status information) to the SVP through Ethernet at 30 messages per second in IEEE C37.118 protocol. The SVP internally aligns the synchrophasor measurements from the PMCUs according to measurement timestamps, calculates the active power measurement of each PMCU, and then feeds these active power measurements to two independent modal analysis function blocks.

Each MA function block accommodates various synchrophasor message rates. If the input synchrophasor message rate exceeds the MA data rate setting, the MA function block downsamples the measurements before the modal analysis calculation. The number of data samples for modal analysis equals the data rate multiplied by the observation window. The sliding window setting specifies the number of new samples necessary for each processing interval of modal analysis. In this test case, each modal analysis function block has the settings in Table I. The MA function blocks calculate the oscillation modes every two seconds (observation window times sliding window divided by 100).

The SVP feeds the results from the two MA function blocks into interarea oscillation mode identification logic. We then set the frequency deviation threshold to 0.01 Hz to identify the common modes. The control logic, shown in Fig. 2, uses the settings from Table II.

TABLE I
MODAL ANALYSIS SETTING

| Estimated # of Modes | Data Rate (Msg/s) | Observation Window (s) | Sliding Window (Percentage) |
|---|---|---|---|
| 15 | 30 | 20 | 10 |

TABLE II
CONTROL LOGIC SETTING

| $SNR_{thre}$ (dB) | $Freq_{low}$ (Hz) | $Freq_{high}$ (HZ) | $Amp_{thre}$ (MW) | $Damp\_Ratio_{thre}$ | DRPU1 (s) |
|---|---|---|---|---|---|
| 80 | 0.1 | 0.8 | 1 | 0.0 | 5 |

Under steady state, Area 1 exports about 425 MW to Area 2 through the intertie transmission lines. We create a small disturbance by reducing the Generator 1 active power output reference from 708 MW to 705 MW. This disturbance triggers an increasing oscillation. Without remedial action, the increasing oscillation will eventually cause the system to collapse. To maintain the stability of the power system, we implemented a simple load-shedding scheme for this test case to increase the damping of the interarea oscillation mode.

Fig. 5 shows the increasing oscillation part of the disturbance. The difference between the two active power measurements in Fig. 5 is a result of line loss. The SVP detects the increasing interarea oscillation mode, asserts the alarm signal about 13 seconds after the change of generation output, and triggers the load-shedding control signal 5 seconds after the alarm signal. Table III lists information about the interarea oscillation model as well as the SNR present 13 seconds after the change of generation output. Table III lists the single interarea oscillation mode that the modal identification logic identified. This mode, with a frequency of about 0.63 Hz, matches the theoretical analysis result [1]. The negative damping ratio indicates an increasing oscillation.

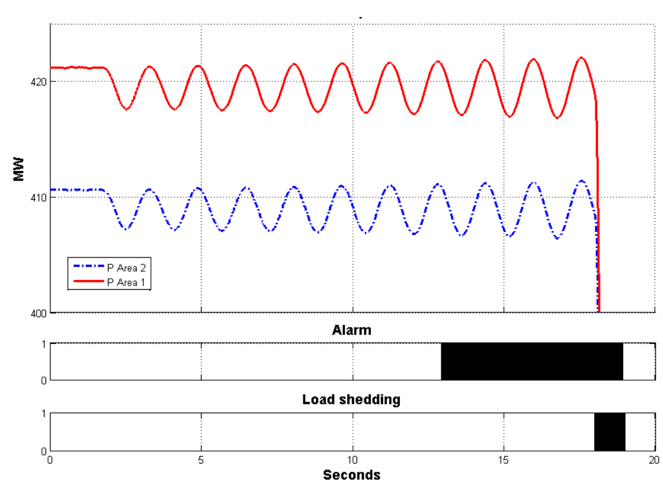


Fig. 5. Increasing interarea oscillation mode

TABLE III
IDENTIFIED INTERAREA OSCILLATION MODE

| | MA1 (P Area 1) | MA2 (P Area 2) |
|---|---|---|
| **SNR (dB)** | 128.0 | 127.5 |
| **Mode_1_Amp (MW)** | 1.04 | 1.03 |
| **Mode_1_Freq (Hz)** | 0.633 | 0.632 |
| **Mode_1_Damp_Ratio** | –0.66% | –0.65% |

The load-shedding scheme reduces the load in Area 2 from 1767 MW to 1350 MW and, as a result, the intertie power transfer decreases from about 420 MW to 204 MW. Fig. 6 shows the active power transfer from Area 1 to Area 2 through the intertie during the event. After the SVP triggers the load shedding, the system slowly settles down to stable condition about 60 seconds later. Fig. 7 shows the mode amplitude of the identified interarea oscillation mode during the disturbance. The small figure in Fig. 7 is an enlarged view of mode amplitude before load shedding occurs. The load-shedding event creates an abrupt change in the interarea power transfer. Therefore, the calculated MA SNR is less than the threshold listed in Table II right after the load shedding, and the SVP reports the amplitude of the interarea oscillation mode as zero. The SNR for both MA instances surpasses the SNR threshold at about the 46th second. Table IV lists the information about the interarea oscillation mode at that moment. The model frequency changes from 0.63 Hz to 0.68 Hz because of the system characteristics change resulting from the load-shedding action. The amplitude of the oscillation mode slowly decreases from 138 MW, and the damping ratio is positive, indicating the oscillation is decaying. The mode frequency and the damping ratio remain constant, and the SNR gradually increases from 85 dB from 150 dB as the event progresses until the oscillation mode eventually dies out.

Real-time digital simulation results with hardware in the test loop demonstrate that the current implementation of the control framework can detect oscillation modes with small amplitude as compared to the base quantity (1 MW out of 420 MW) and provide real-time detection of oscillation modes. Decision and control logic schemes can incorporate modal analysis results as reliable inputs to alarm for unstable oscillations and trigger remedial action schemes to mitigate unstable interarea oscillations.

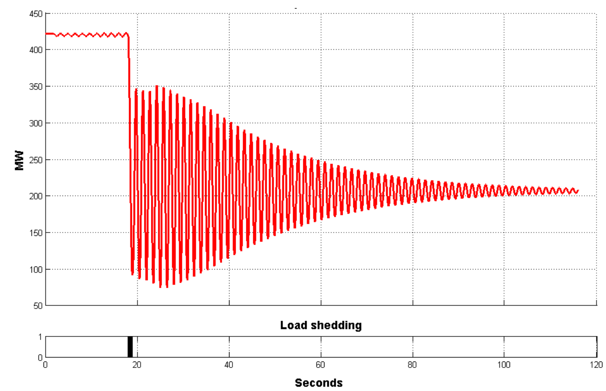


Fig. 6. Interarea active power transfer throughout the event

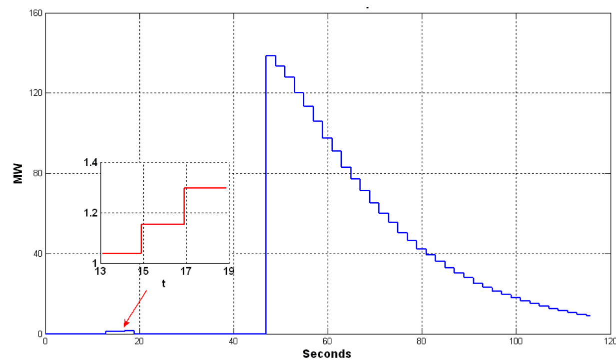


Fig. 7. Identified interarea oscillation mode amplitude throughout the event

TABLE IV
THE FIRST IDENTIFIED INTERAREA OSCILLATION MODE
AFTER THE LOAD-SHEDDING ACTION

| | MA1 (Area 1) | MA2 (P Area 2) |
|---|---|---|
| **SNR (dB)** | 84.2 | 84.9 |
| **Mode_1_Amp (MW)** | 138.6 | 136.5 |
| **Mode_1_Freq (Hz)** | 0.685 | 0.685 |
| **Mode_1_Damp_Ratio** | 1.03% | 1.01% |

## V. CONCLUSIONS

This paper presents a SIPS to detect and mitigate unstable power system interarea oscillations. The system includes a SVP that acquires synchrophasor measurements from PMCUs located at different geographical areas, performs modal analysis, and executes custom logic in real time. The SVP implements modified Prony analysis that allows for more data samples within the observation window than are available in traditional Prony analysis. This increased observation window improves the resolution of the modal frequency estimation. Custom logic includes logic to identify interarea oscillations and trigger remedial actions. Hardware-in-the-loop test results demonstrate that the present implementation of this SIPS can detect unstable interarea oscillation modes with small amplitude early and initiate automatic remedial actions to mitigate unstable oscillations in real time.

## VI. Appendix A

TABLE V
GENERATOR OUTPUTS IN THE BASE CASE

| | Gen 1 | Gen 2 | Gen 3 | Gen 4 |
|---|---|---|---|---|
| **P (MW)** | 707 | 720 | 704 | 695 |
| **Q (MVAR)** | 101 | 137 | 77 | 81 |

TABLE VI
BASE CASE LOAD

| | Load Type | P | Q |
|---|---|---|---|
| **Load 1** | Constant P & Q | 967 MW | 100 MVAR |
| **Load 2** | Constant P & Q | 1767 MW | 100 MVAR |

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## VIII. Biographies

**Yanfeng Gong** received his BSEE from Wuhan University, Wuhan, China in 1998, his MSEE from Michigan Technological University, Houghton, MI, in 2002, and his PhD in Electrical Engineering from Mississippi State University in 2005. He is currently working with Schweitzer Engineering Laboratories, Inc. as a research engineer in Pullman, Washington. He is a member of IEEE.

**Armando Guzmán** received his BSEE with honors from Guadalajara Autonomous University (UAG), Mexico, in 1979. He received a diploma in fiber-optics engineering from Monterrey Institute of Technology and Advanced Studies (ITESM), Mexico, in 1990, and his MSEE from the University of Idaho, USA, in 2002. He served as regional supervisor of the Protection Department in the Western Transmission Region of the Federal Electricity Commission (the electrical utility company of Mexico) in Guadalajara, Mexico for 13 years. He lectured at UAG and the University of Idaho in power system protection and power system stability. Since 1993, he has been with Schweitzer Engineering Laboratories, Inc. in Pullman, Washington, where he is presently Research Engineering Manager. He holds several patents in power system protection and metering. He is a senior member of IEEE and has written several technical papers.


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