Stator Thermal Time Constant

Jon Steinmetz, Subhash C. Patel, and Stanley E. Zocholl
Schweitzer Engineering Laboratories, Inc.

This paper was presented at the 49th Annual Industrial & Commercial Power Systems Technical Conference and can be accessed at: http://dx.doi.org/10.1109/ICPS.2013.6547350.

For the complete history of this paper, refer to the next page.

Presented at the
49th Annual Industrial & Commercial Power Systems Technical Conference
Stone Mountain, Georgia
April 30–May 3, 2013

Previously published in
SEL *Journal of Reliable Power*, Volume 3, Number 1, March 2012

Originally presented at the
37th Annual Western Protective Relay Conference, October 2010

Stator Thermal Time Constant

Jon Steinmetz
Member, IEEE
Schweitzer Engineering Laboratories, Inc.
128 Azalea Drive
Ringgold, GA 30736 USA
jon_steinmetz@selinc.com

Subhash C. Patel
Senior Member, IEEE
Schweitzer Engineering Laboratories, Inc.
671 Moore Road, Suite 200
King of Prussia, PA 19406 USA
subhash_patel@selinc.com

Stanley E. Zocholl
Life Fellow, IEEE
Schweitzer Engineering Laboratories, Inc.
2350 NE Hopkins Court
Pullman, WA 99163 USA

***Abstract*—The thermal model providing motor overload protection is derived from the first order differential equation for heat rise due to current in a conductor. Only the stator thermal time constant and the service factor are the required settings. The thermal model utilizes the full thermal capacity of the motor and allows current swings and cyclic overloads that would trip conventional overcurrent protection but do not actually overheat the motor. Four examples of thermal limit curves and their equations are used to discuss the varying plotting practices in use. The paper also includes a method to calculate the stator thermal time constant using two points read from the overload curve when not available from motor data.**

***Index Terms*—Cyclic overload, inverse overcurrent curve, motor thermal model, service factor, thermal limit curve, time constant**

I. Instruction

This paper explains the use of thermal limit curves for motor thermal protection as distinguished from the use of overcurrent characteristics for overcurrent protection. Fig. 1 shows the running overload curve of a 2027 hp, 6600 V PA fan motor.

The curve in Fig. 1 resembles an inverse overcurrent relay as defined in IEEE C37.112 *Standard Inverse-Time Characteristic Equations for Overcurrent Relays* with the equation:

$$t(I) = \frac{A}{\left(\frac{I}{I_P}\right)^2 - 1} \tag{1}$$

where:

I is current.

I_P is the pickup current.

A is a constant.

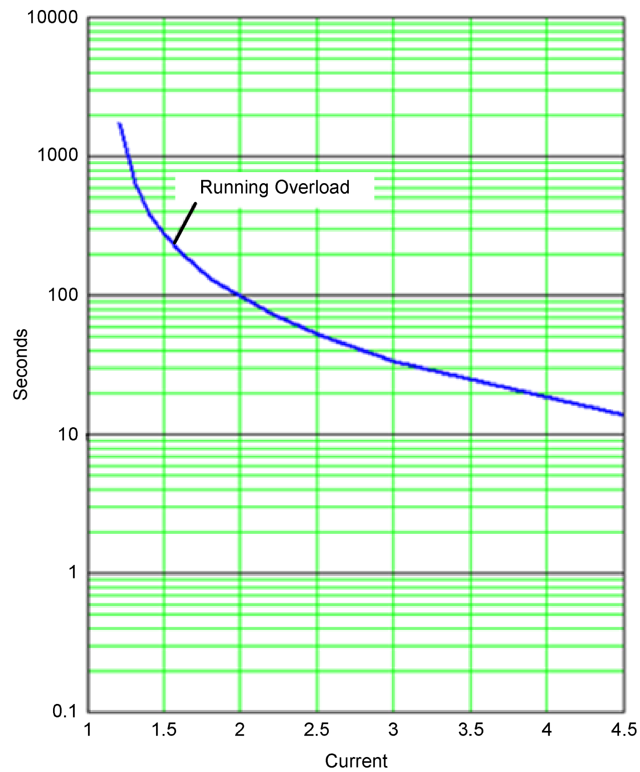

Fig. 1. 2027 hp, 6600 V Motor Running Thermal Limit Curve

To maintain coordination with overcurrent relays even with varying current, the dynamics would be implemented according to the integral equation:

$$\int_0^{T_0} \frac{1}{t(I)} dt = 1 \tag{2}$$

In Fig. 2, the inverse characteristic with the constant A = 190 superimposed on the running overload curve is an almost exact fit. It shows that a long-time inverse-time overcurrent relay applied with minimal coordination margin can provide conservative overcurrent protection for motor overload.

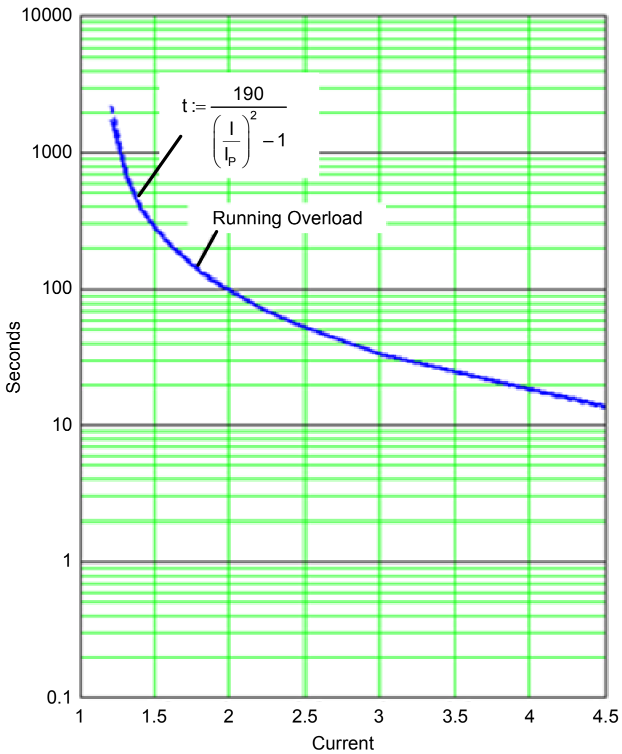

Fig. 2. Running Overload With Superimposed Overcurrent Curve

II. THE THERMAL LIMIT CURVE

However, the running overload curve of Fig. 1, rather than an overcurrent curve, is a thermal limit and has the equation:

$$t = TC \bullet \ln\left(\frac{I^2 - I_0^2}{I^2 - SF^2}\right) \tag{3}$$

where:

t is the time to reach the limiting temperature.

TC is the stator thermal time constant.

I is the current in per unit of rated full load.

I_0 is the preload current.

SF is the service factor (maximum continuous current).

In this case, the thermal time constant is 3720 seconds, the service factor is 1.15, and the preload I_0 is 1.12. The curve is derived from the first order thermal model for heating due to current in a conductor, as derived in the annex. It is the locus of time-current points that produce the limiting temperature caused by the rated maximum continuous current SF. In this equation, *t* is the time to reach the limiting temperature, starting from the preload temperature.

Consequently, where the overcurrent curve is fixed, the thermal limit curve shows only one of many possible curves, depending on the preload current for which it is plotted. Fig. 3 shows the curves for a range of preload values.

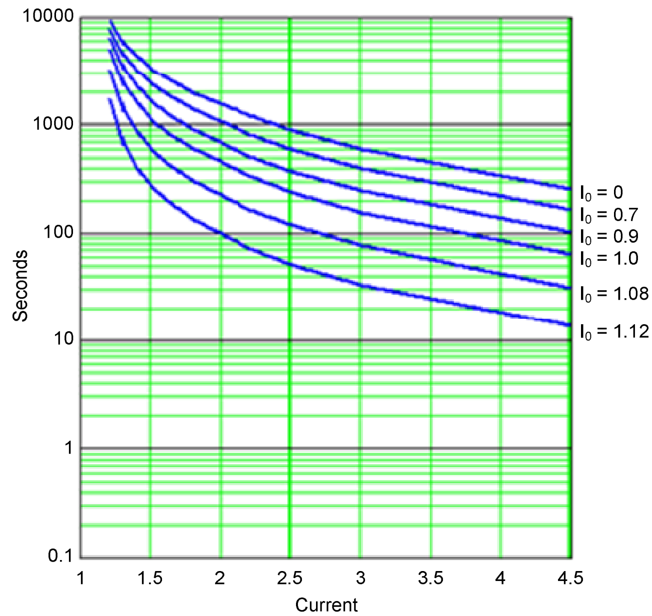

Fig. 3. Thermal Limit Curve for a Range of Preload Current

The stator thermal model is easily implemented in a microprocessor motor relay as:

$$U_n = I^2 \bullet \frac{\Delta t}{\tau} + \left(1 - \frac{\Delta t}{\tau}\right) \bullet U_{n-1} \tag{4}$$

where:

U_n is temperature in units I^2 at current sample *n*.

Δt is the sample time increment.

τ is the time constant.

U_{n-1} is the temperature at the previous sample.

Equation (4) calculates the temperature in units of I^2. The plot in Fig. 4 shows 1.15 per-unit current applied for 167 minutes and then stepped down to 1.0 per-unit current. The temperature rises exponentially from an initial temperature of one per unit and then decays back to the original temperature. For overload protection, the thermal model settings are simply the thermal time constant (3720 seconds) and the service factor (1.15).

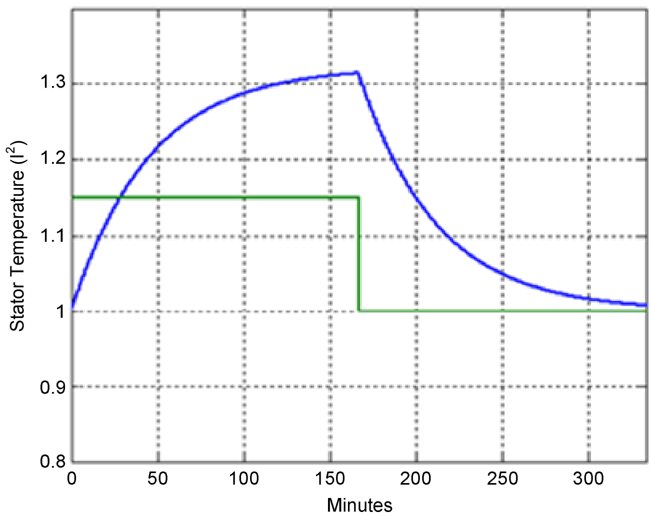

Fig. 4. Stator Temperature Responding to a Step of Current

The thermal model has the advantage of using the full thermal capacity of the motor, allowing transient current swings and cyclic overloads that would trip the overcurrent relay but do not actually overheat the motor. Fig. 5 shows the temperature response U of the thermal model to an overload alternating between 1.4 and 0.5 per-unit current every 12 minutes. Fig. 6 shows that the overcurrent relay trips in 6.57 minutes for the cyclic overload that does not overheat the motor.

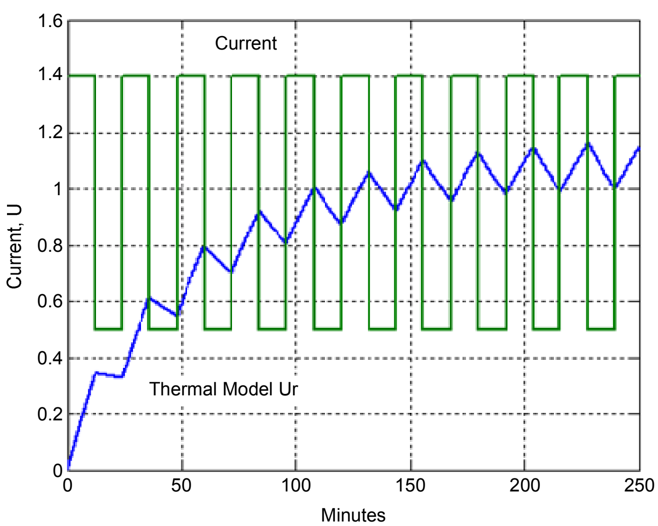

Fig. 5. Thermal Model Response to a Cyclic Overload

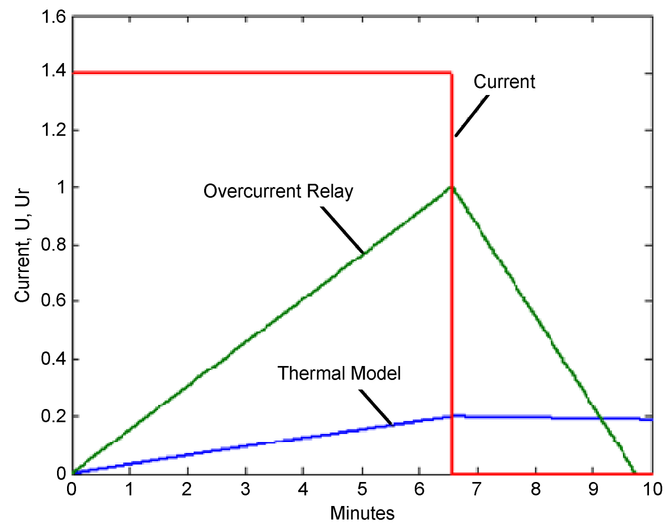

Fig. 6. Overcurrent Relay Tripping for the Cyclic Overload

III. Overload Curve Examples

Four examples of stator thermal limit curves from three different manufacturers are shown in the following figures. The equations for each curve are included, and the time constant in each case was obtained from motor data. These examples show two different plotting practices.

The service factor is the asymptote of the curve in Fig. 7 and Fig. 8. However, the curve of Fig. 9 has a much higher asymptote at 1.65 per unit. It shows the overload for the highest current at which the motor can run without stalling and is not an indication of constant overload capability. The asymptote in Fig. 10 also exceeds the service factor.

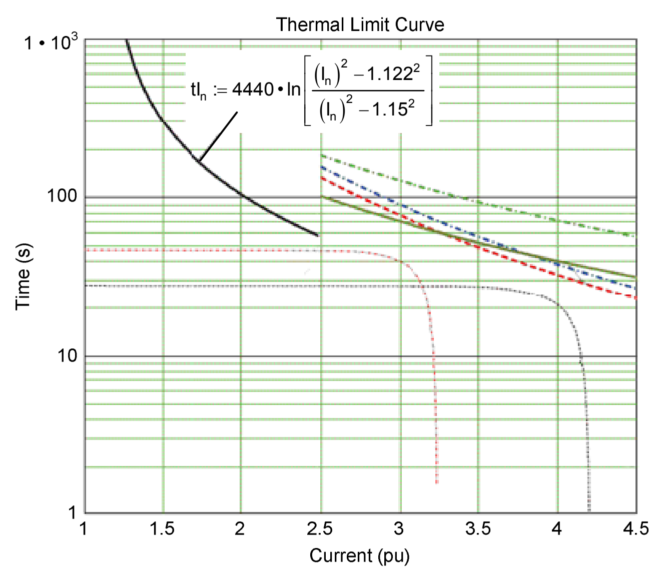

Fig. 7. Thermal Limit Curve for a WEG 20421 hp ID Fan Motor

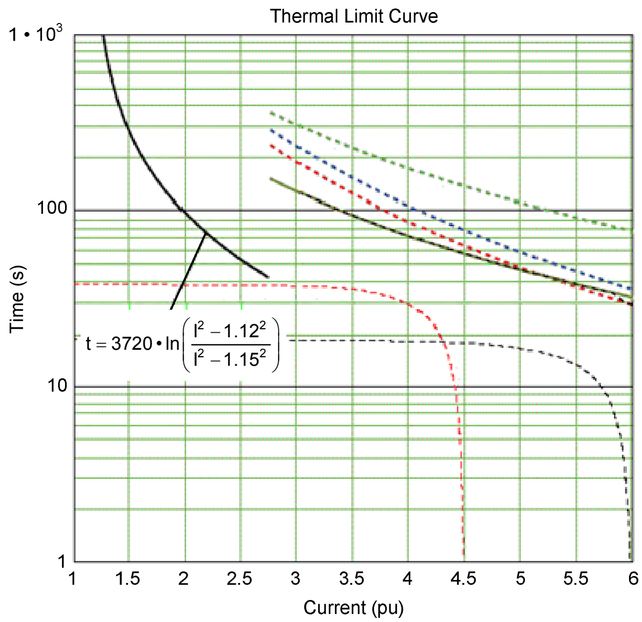

Fig. 8. Thermal Limit Curve for a WEG 2027 hp PA Fan Motor

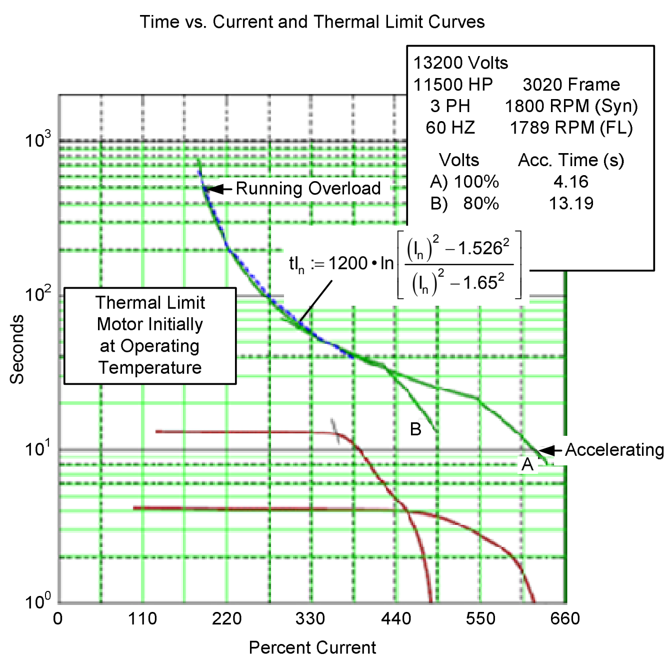

Fig. 9. Thermal Limit Curve for a TECO 1150 hp Motor

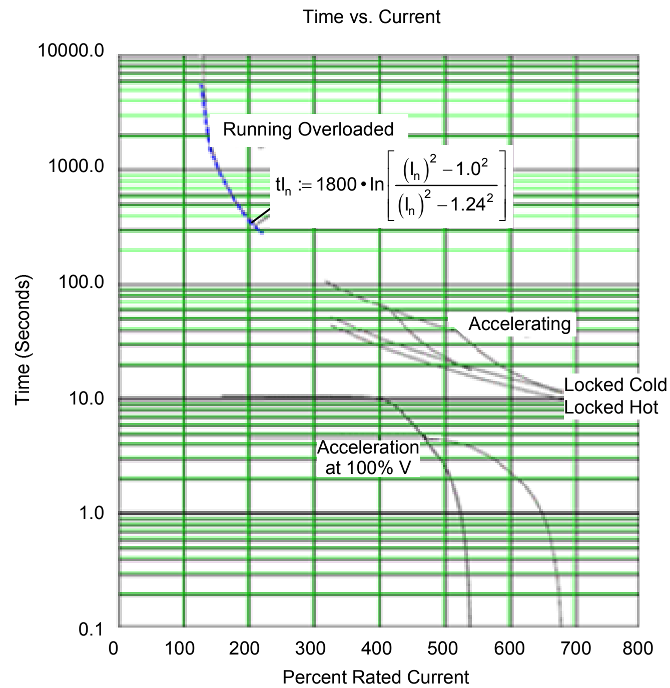

Fig. 10. Thermal Limit Curve for a Siemens 2250 hp Motor

This type of overload plot appears in IEEE 620 *Guide for the Presentation of Thermal Limit Curves for Squirrel Cage Induction Machines*. However, the protection settings remain the time constant and service factor, the maximum rated continuous current.

IV. CALCULATING THE TIME CONSTANT

When the data are not available, the time constant can be calculated using two points read from the thermal limit curve. Only one value of the preload current I_0 will give the same time constant TC in the following pair of equations:

$$TC1 = \frac{t_1}{\ln\left(\frac{I_1^2 - I_0^2}{I_1^2 - SF^2}\right)} \quad (5)$$

$$TC2 = \frac{t_2}{\ln\left(\frac{I_2^2 - I_0^2}{I_2^2 - SF^2}\right)} \quad (6)$$

$(I_1, t_1) = (1.5, 263.6)$ and $(I_2, t_2) = (2.5, 51.06)$ are the coordinates of the points read from the curve of Fig. 1. Inserting these values in (5) and (6) with SF = 1.15 yields TC = 3720 for $I_0 = 1.12$. The specific preload I_0 occurs where the ratio of TC1 to TC2 is 1.0 in the plot of the ratio as a function of I_0, as shown in Fig. 11.

$$TC1 = \frac{263.6^2}{\ln\left(\frac{1.5^2 - I_0^2}{1.5^2 - SF^2}\right)} = 3720$$

$$TC2 = \frac{51.06}{\ln\left(\frac{2.5^2 - I_0^2}{2.5^2 - SF^2}\right)} = 3720$$

Therefore, the equation of the curve in Fig. 1 is:

$$t = 3720 \bullet \ln\left(\frac{I^2 - 1.12^2}{I^2 - 1.15^2}\right) \tag{7}$$

Fig. 11. The Ratio of TC1 to TC2 as a Function of Values of Preload I_0

V. Conclusion

1. Motor overload curves are derived from the first order thermal model for heating due to current in a conductor. It is the locus of time-current points that produce the limiting temperature.
2. Overload protection is provided in the form of a first order thermal model, where the time constant and the service factor (SF) are settings.
3. A long-time inverse-time overcurrent relay provides conservative overload protection. However, thermal protection provides full use of motor thermal capacity, allowing transient current and cyclic overloads that would trip the overcurrent relay but do not overheat the motor.
4. When the time constant is unavailable, it can be calculated using two points read from the thermal limit curve.

VI. Annex – First Order Thermal Model

The first order thermal model is derived as follows:

$$\theta = \theta_W - \theta_A \tag{8}$$

where:

θ_W is the winding temperature.

θ_A is the ambient temperature.

The rate of increase of the temperature is given by the equation expressing the thermal equilibrium.

$$\text{Power Supplied} - \text{Losses} = C_s m \frac{d\theta_W}{dt} = C_s m \frac{d\theta}{dt} \tag{9}$$

In this equation, C_s is the specific heat of the winding and *m* is the mass. The specific heat is the amount of energy needed to raise one kilogram of that material one degree centigrade. The losses or the quantity of heat transferred to the surrounding environment is expressed as:

$$\text{Losses} = \frac{\theta_W - \theta_A}{R} = \frac{\theta}{R} \tag{10}$$

where:

R is the thermal resistance in °C/watt.

Equation (9) can be otherwise expressed as:

$$I^2 r - \frac{\theta}{R} = C_s m \frac{d\theta}{dt} \tag{11}$$

or

$$I^2 r \bullet R = C_s m \bullet R \frac{d\theta}{dt} + \theta \tag{12}$$

The mass *m* multiplied by the specific heat C_s is known as C, the thermal capacity of the system with units of joules/°C. It represents the amount of energy in joules required to raise the system temperature by one degree centigrade. The product of the thermal resistance R and the thermal capacitance C has units of seconds and represents the thermal time constant:

$$\tau = R \bullet m \bullet C_s \tag{13}$$

The fundamental equation (12) can be expressed in a simpler form:

$$I^2 = C_s m \bullet R \bullet \left(\frac{1}{r \bullet R} \bullet \frac{d\theta}{dt} \right) + \frac{\theta}{r \bullet R} \tag{14}$$

$$\tau = C_s m \bullet R \tag{15}$$

let

$$U = \frac{\theta}{r \bullet R} \tag{16}$$

and

$$\frac{dU}{dt} = \frac{1}{r \bullet R} \bullet \frac{d\theta}{dt} \tag{17}$$

Therefore, the first order thermal model equation becomes the simple form:

$$I^2 = \tau \frac{dU}{dt} + U \tag{18}$$

The solution of the first order equation is:

$$U = I^2 \bullet \left(1 - e^{\frac{-t}{\tau}} \right) \tag{19}$$

With an initial value U_0:

$$U = I^2 \bullet \left(1 - e^{\frac{-t}{\tau}} \right) + U_0 e^{\frac{-t}{\tau}} \tag{20}$$

Solving the equation for *t* gives the time to reach a specific temperature in units of I^2:

$$t = \tau \bullet \ln \left(\frac{I^2 - U_0}{I^2 - U} \right) \tag{21}$$

Since the temperature is in units of I^2, U and U_0 can be expressed as values of current squared:

$$t = \tau \bullet \ln \left(\frac{I^2 - I_0^2}{I^2 - I_{max}^2} \right) \tag{22}$$

When using (20) to calculate U over a small time increment Δt, the exponentials can be replaced with the first two terms of the infinite series as follows:

$$e^{\frac{-\Delta t}{\tau}} = \left(1 - \frac{\Delta t}{\tau} \right) \tag{23}$$

Substituting (23) in (20) gives:

$$U_n = I^2 \bullet \left(1 - \left(1 - \frac{\Delta t}{\tau} \right) \right) + U_{n-1} \bullet \left(1 - \frac{\Delta t}{\tau} \right) \tag{24}$$

This incremental form of the equation is ideal for use in the processor for the continuous real-time calculation of temperature:

$$U_n = \frac{I^2}{\tau} \Delta t + \left(1 - \frac{\Delta t}{\tau} \right) \bullet U_{n-1} \tag{25}$$

where:

U_n is the temperature expressed in units of I^2 at sample *n*.

U_{n-1} is the temperature expressed in units of I^2 at the previous sample.

Electrical engineers find it helpful to visualize the thermal model as an electrical analog circuit. The first order equation of the thermal model has the same form as the equation expressing the voltage rise in an electrical RC circuit, as shown in Fig. 12.

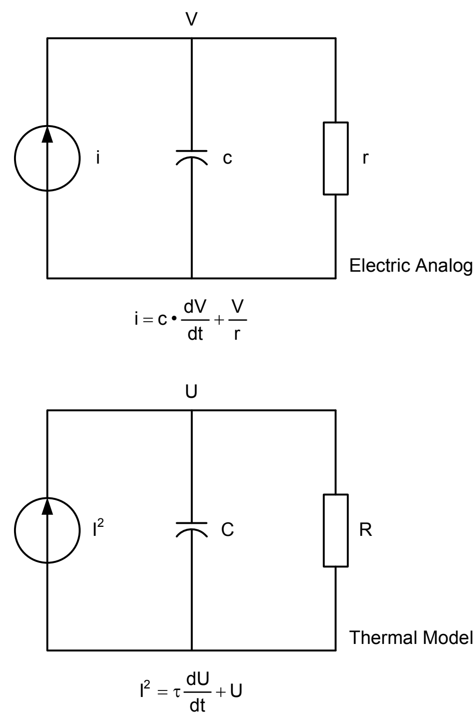

Fig. 12. The Electrical Analog Circuit of the Thermal Model

In Fig. 12, the lowercase letters are used to identify the electrical parameters. In the circuit, the voltage V is the analog of the temperature U; the constant current *i* is numerically equal to the current squared. The thermal resistance R and thermal capacitance C are the direct analogs of the electrical resistance *r* and the electrical capacitance *c*.

VII. References

[1] S. E. Zocholl, *AC Motor Protection*, 2nd ed., Schweitzer Engineering Laboratories, Inc., ISBN 0-9725026-1-0, 2003.

[2] S. E. Zocholl, "Tutorial: From the Steinmetz Model to the Protection of High Inertia Drives," proceedings of the 33rd Annual Western Protective Relay Conference, Spokane, WA, October 2006.

[3] *IEEE Guide for the Presentation of Thermal Limit Curves for Squirrel Cage Induction Machines*, IEEE Standard 620.

[4] IEC 60255-8 1990, *Electrical Relays – Part 8: Thermal Electrical Relays*.

[5] S. E. Zocholl and G. Benmouyal, "Using Thermal Limit Curves to Define Thermal Models of Induction Motors," proceedings of the 28th Annual Western Protective Relay Conference, Spokane, WA, October 2001.

[6] S. E. Zocholl and G. Benmouyal, "On the Protection of Thermal Processes," *IEEE Transactions on Power Delivery*, Vol. 20, Issue 2, Part II, pp. 1240–1246, April 2005.

VIII. Biographies

Jon Steinmetz graduated from West Virginia Institute of Technology with a B.S. in electrical engineering and is a registered professional engineer in the state of West Virginia. He joined Schweitzer Engineering Laboratories, Inc. (SEL) as a field application engineer specializing in industrial protection and control applications. Prior to joining SEL, Mr. Steinmetz worked in protection and control positions with American Electric Power and Union Carbide Corporation.

Subhash C. Patel received his B.S.E.E. and B.S.M.E. degrees from the M. S. University, Baroda, India, in 1965 and 1966, respectively. He worked for Brown Boveri Company in India before coming to the USA in late 1967. He received the M.S.E.E. degree from the University of Missouri - Rolla in 1969 and joined Illinois Power Company in Decatur, Illinois, where he was primarily responsible for power system protection. He was with GE from 1979 to 1999, during which he had various assignments in the field of protection and control as well as gas turbine package power plants. In 1999, Mr. Patel joined Schweitzer Engineering Laboratories, Inc. as a field application engineer and currently is a principal power engineer in Pennsylvania. He is a Life Senior Member of IEEE, a member of SC-J and several working groups of PSRC, a registered professional engineer in the states of New Hampshire and Illinois, and an author of several protective relay conference papers.

Stanley E. (Stan) Zocholl has a B.S. and M.S. in Electrical Engineering from Drexel University. He is an IEEE Life Fellow and a member of the Power Engineering Society and the Industrial Application Society. He is also a member of the Power System Relaying Committee. He joined Schweitzer Engineering Laboratories, Inc. in 1991 in the position of Distinguished Engineer. He was with ABB Power T&D Company Allentown (formerly ITE, Gould BBC) since 1947 where he held various engineering positions, including Director of Protection Technology. His biography appears in Who's Who in America. He holds over a dozen patents associated with power system protection using solid state and microprocessor technology and is the author of numerous IEEE and protective relay conference papers. He received the Power System Relaying Committee's Distinguished Service Award in 1991. He was the Chairman of PSRCW G J2 that completed the AC Motor Protection Tutorial. He is the author of two books, *AC Motor Protection*, second edition, ISBN 0-9725026-1-0, and *Analyzing and Applying Current Transformers,* ISBN 0-9725026-2-9.

Previously presented at the 49th Annual Industrial & Commercial Power Systems Technical Conference, Stone Mountain, GA, April 2013.

20130129 • TP6445