Calculating Intermediate Faults
in Underground Cables

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Abstract—**Short-circuit calculations are extremely important in the application and setting of protective relays and the analysis of power system operations.**

This paper illustrates the calculation of fault currents and voltages during intermediate faults in underground cables, taking into consideration the cable sheath bonding and grounding method. The compensated ground loop impedance for faults along the cable is nonlinear with respect to fault distance when the cable sheaths are cross-bonded and grounded at both cable ends. This makes the application of ground distance relay elements for cable protection complex and undesirable.

This paper provides numerical examples of intermediate fault analysis in underground cables to help protection engineers study similar or more advanced power system faults. This paper demonstrates the importance of phase frame of reference calculations in fault analysis, relay settings considerations, and an overall understanding of power system behavior. In addition, this paper shows the comparison of fault calculation results between Mathcad® and an Electromagnetic Transients Program steady-state solution.

I. INTRODUCTION

Protection engineers frequently use phase and ground distance elements in directional comparison schemes for cable protection. They also typically use Zone 1 distance for instantaneous tripping of cable faults and Zone 2 time-delayed tripping for backup cable protection. Distance relay element application for cable protection requires a good knowledge of cable electrical parameters and their dependence on sheath bonding and grounding and a good understanding of available relay technology.

Note that in overhead transmission lines, the positive- and zero-sequence line impedances (Z_{1L} and Z_{0L}) are proportional to line length. However, this is not true for underground cables, where the zero-sequence impedance may be nonlinear with respect to distance along the cable. The zero-sequence compensation factor (k_0) for solid and cross-bonded cables is not constant for internal cable faults, and it depends on the location of the fault along the cable circuit. Because ground distance relays use a single value of k_0, the compensated loop impedance displays nonlinear behavior.

The application of ground distance relays for underground cable protection is a challenging problem because the compensated ground loop impedance depends on the ground fault current return path. The ground fault current return path depends on the cable sheath bonding and grounding method, any other conducting paths in parallel with the cable, and the fault point along the cable. Understanding how the cable grounding method affects the apparent impedances of ground

distance relays is fundamental to underground cable protection [1].

Most faults in underground single-conductor cables involve ground. For that reason, it is important to calculate the fault quantities during intermediate faults along the cable in order to calculate the apparent impedances seen by distance relays.

Short-circuit programs provide the bus voltages and line currents, in the phase and sequence domain, for simple balanced and unbalanced short circuits in the network under study. In many situations, the protection engineer has to analyze more complex fault types that are not handled in standard short-circuit programs and typically not taught in a four-year electrical engineering curriculum [2]. Such is the case with the analysis of intermediate faults in underground cables with different methods of sheath grounding and bonding. Hence, the protection engineer has to resort to more advanced programs, such as the Electromagnetic Transients Program (EMTP), in order to provide answers to the problems at hand or use steady-state phase frame of reference methods using any engineering calculation software (e.g., Mathcad®, as discussed in this paper).

This paper provides a method for the calculation of intermediate short-circuit faults in underground cables, taking into consideration sheath bonding and grounding. The short-circuit method proposed in this paper is implemented in Mathcad and uses the phase frame of reference approach instead of the symmetrical component method. The phase frame of reference approach easily handles the complexity of cable core-to-core and core-to-sheath coupling, cable cross-bonding, and the different types of cable sheath grounding. This paper shows the importance of phase frame of reference calculations in solving complex power system faults in underground cables.

II. INTERMEDIATE FAULTS IN UNDERGROUND CABLES WITH SHEATHS GROUNDED AT BOTH CABLE ENDS

The study of intermediate faults in underground cables using symmetrical component theory is quite complex. One limitation of symmetrical component theory is the assumption that power system element impedances are balanced. This is not true in underground cables because of the different approaches used for cable sheath bonding and grounding. Another difficulty in applying symmetrical component theory is the need to retain the sheaths, including their transpositions and grounding along the cable path, to properly study intermediate faults along the cable. All of these difficulties are

overcome by using the phase frame of reference approach and a good mathematical programming software package.

To analyze intermediate faults in underground cables, we create a generalized fault model of a single underground cable section with its sheaths grounded at both cable ends. This helps us understand how to create the generalized fault model. Next, we introduce additional complexities, such as multiple cable sections with sheath or core transpositions, core-to-sheath or core-to-sheath-to-ground faults, and faults through an impedance.

Fig. 1 shows a one-line diagram of an intermediate fault in a three-phase single-core underground cable section with the sheaths grounded at both ends of the cable. The sheaths are connected together at each cable end and grounded through resistances for the purpose of creating a generalized fault model. In the generalized fault model, we can vary the sheath grounding impedance from zero to an infinite value to simulate various types of grounding at one or both ends of the cable. In addition, three impedances at the intermediate fault point model core-to-sheath, core-to-ground, and core-to-sheath-to-ground faults.
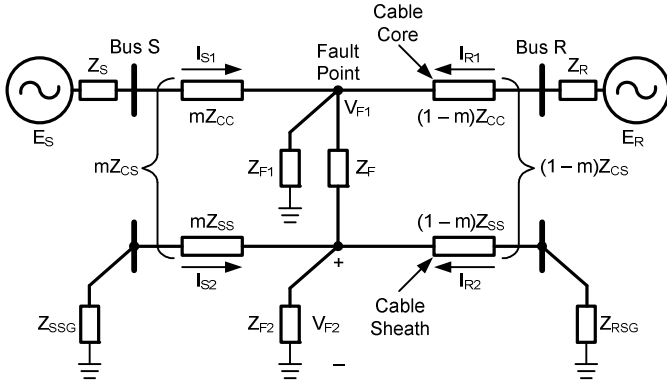

Fig. 1. Generalized underground cable intermediate fault model

The following system of equations describes the fault model of the network shown in Fig. 1:

$$E_S = \left(Z_S + mZ_{CC}\right)I_{S1} + mZ_{CS}I_{S2} + V_{F1} \qquad (1)$$

$$E_R = \left[Z_R + \left(1-m\right)Z_{CC}\right]I_{R1} + \left(1-m\right)Z_{CS}I_{R2} + V_{F1} \qquad (2)$$

$$0 = mZ_{CS}^{T}I_{S1} + \left(Z_{SSG} + mZ_{SS}\right)I_{S2} + V_{F2} \qquad (3)$$

$$0 = \left(1-m\right)Z_{CS}^{T}I_{R1} + \left[Z_{RSG} + \left(1-m\right)Z_{SS}\right]I_{S2} + V_{F2} \qquad (4)$$

$$0 = I_{S1} + I_{R1} - \left(Z_{F1}^{-1} + Z_{F}^{-1}\right)V_{F1} + Z_{F}^{-1}V_{F2} \qquad (5)$$

$$0 = I_{S2} + I_{R2} + Z_{F}^{-1}V_{F1} - \left(Z_{F2}^{-1} + Z_{F}^{-1}\right)V_{F2} \qquad (6)$$

where:

E_S = Bus S (sending-end) source voltages, 3x1 vector.

E_R = Bus R (receiving-end) source voltages, 3x1 vector.

Z_S = sending-end source impedance, 3x3 matrix.

Z_R = receiving-end source impedance, 3x3 matrix.

Z_{CC} = cable core-to-core series impedance, 3x3 matrix.

Z_{CS} = cable core-to-sheath series impedance, 3x3 matrix.

Z_{SS} = cable sheath-to-sheath series impedance, 3x3 matrix.

Z_{CS}^{T} = cable core-to-sheath series impedance transpose, 3x3 matrix.

I_{S1} = sending-end core currents, 3x1 vector.

I_{S2} = sending-end sheath currents, 3x1 vector.

I_{R1} = receiving-end core currents, 3x1 vector.

I_{R2} = receiving-end sheath currents, 3x1 vector.

V_{F1} = intermediate point core-to-ground voltage, 3x1 vector.

V_{F2} = intermediate point sheath-to-ground voltage, 3x1 vector.

Z_{F1} = intermediate point core-to-ground fault impedance, 3x3 matrix.

Z_{F2} = intermediate point sheath-to-ground fault impedance, 3x3 matrix.

Z_{F} = intermediate point core-to-sheath fault impedance, 3x3 matrix.

Z_{SSG} = sending-end sheath-to-ground impedance, 3x3 matrix.

Z_{RSG} = receiving-end sheath-to-ground impedance, 3x3 matrix.

Next, we rewrite (1) through (6) in compact matrix form.

$$E = AX \qquad (7)$$

where:

$$E = \begin{bmatrix} E_S \\ E_R \\ E_0 \\ E_0 \\ E_0 \\ E_0 \end{bmatrix} \qquad (8)$$

$$X = \begin{bmatrix} I_{S1} \\ I_{S2} \\ I_{R1} \\ I_{R2} \\ V_{F1} \\ V_{F2} \end{bmatrix} \qquad (9)$$

Each element in Matrices E and X is a 3x1 column vector. Matrix A is an 18x18 square matrix made up of cable series impedances and fault admittances.

From (7), we calculate the core and sheath currents and the voltages at the fault point in Vector X, knowing Vector E and Matrix A. Next, we calculate the voltages at the sending- and receiving-end buses, knowing the source voltages and the core and sheath currents calculated in the previous step.

Having knowledge of all the phase voltages and currents at the two ends of the cable (Buses S and R), we can then calculate the sequence component voltages and the ground loop impedances as seen from Buses S and R.

$$A = \begin{bmatrix} (Z_S + mZ_{CC}) & mZ_{CS} & O & O & I & O \\ O & O & [Z_{SR} + (1-m)Z_{CC}] & (1-m)Z_{CS} & I & O \\ mZ_{CS}^T & (Z_{SSG} + mZ_{SS}) & O & O & O & I \\ O & O & (1-m)Z_{CS}^T & [Z_{RSG} + (1-m)Z_{SS}] & O & I \\ I & O & I & O & -(Z_{F1}^{-1} + Z_F^{-1}) & Z_F^{-1} \\ O & I & O & I & Z_F^{-1} & -(Z_{F2}^{-1} + Z_F^{-1}) \end{bmatrix} \quad (10)$$

Matrix A is shown in (10), and each of its elements is described further in Section II, Subsection C.

A. Formulation of the E Column Vector

Each of the elements in Matrix E is a 3x1 column vector. E_S and E_R consist of a balanced set of sending- and receiving-end three-phase voltages. E_0 is a zero column vector with dimensions of 3x1, as shown in (11).

$$E_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (11)$$

The sending- and receiving-end source voltages are:

$$E_S = e_S B \quad (12)$$

$$E_R = e_R B \quad (13)$$

where:

$$B = \begin{bmatrix} 1 & a^2 & a \end{bmatrix}^T \quad (14)$$

$$e_S = kV_{LN} \quad (15)$$

$$e_R = kV_{LN}e^{j\delta} \quad (16)$$

$$a = e^{j120} = -0.5 + j0.8660254 \quad (17)$$

In (15) and (16), kV_{LN} is the system line-to-neutral voltage and δ is the angle difference between the sending- and receiving-end source voltages. In (17), a is an operator with a unit magnitude and an angle of 120 degrees that causes a rotation of 120 degrees in the counterclockwise direction.

B. Formulation of the X Column Vector

Equations (18) through (23) show the elements of the X column vector in (7).

$$I_{S1} = \begin{bmatrix} I_{S\_ca} \\ I_{S\_cb} \\ I_{S\_cc} \end{bmatrix} \quad (18)$$

$$I_{S2} = \begin{bmatrix} I_{S\_sha} \\ I_{S\_shb} \\ I_{S\_shc} \end{bmatrix} \quad (19)$$

$$I_{R1} = \begin{bmatrix} I_{R\_ca} \\ I_{R\_cb} \\ I_{R\_cc} \end{bmatrix} \quad (20)$$

$$I_{R2} = \begin{bmatrix} I_{R\_sha} \\ I_{R\_shb} \\ I_{R\_shc} \end{bmatrix} \quad (21)$$

$$V_{F1} = \begin{bmatrix} V_{F1\_a} \\ V_{F1\_b} \\ V_{F1\_c} \end{bmatrix} \quad (22)$$

$$V_{F2} = \begin{bmatrix} V_{F2\_a} \\ V_{F2\_b} \\ V_{F2\_c} \end{bmatrix} \quad (23)$$

C. Formulation of Matrix A

Each element in Matrix A is a 3x3 submatrix. The elements of Matrix A are shown in (24) through (37).

$$O = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (24)$$

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (25)$$

The series impedance matrix of a single-core three-phase underground cable is:

$$Z_C = \begin{bmatrix} Z_{CC} & Z_{CS} \\ Z_{SC} & Z_{SS} \end{bmatrix} = \begin{bmatrix} Z_{CC} & Z_{CS} \\ Z_{CS}^T & Z_{SS} \end{bmatrix} \quad (26)$$

where:

$$Z_{CC} = \begin{bmatrix} z_{ca\_ca} & z_{ca\_cb} & z_{ca\_cc} \\ z_{cb\_ca} & z_{cb\_cb} & z_{cb\_cc} \\ z_{cc\_ca} & z_{cc\_cb} & z_{cc\_cc} \end{bmatrix} \quad (27)$$

$$Z_{CS} = \begin{bmatrix} z_{ca\_sa} & z_{ca\_sb} & z_{ca\_sc} \\ z_{cb\_sa} & z_{cb\_sb} & z_{cb\_sc} \\ z_{cc\_sa} & z_{cc\_sb} & z_{cc\_sc} \end{bmatrix} \quad (28)$$

$$Z_{SC} = Z_{CS}^T \quad (29)$$

$$Z_{SS} = \begin{bmatrix} z_{sa\_sa} & z_{sa\_sb} & z_{sa\_sc} \\ z_{sb\_sa} & z_{sb\_sb} & z_{sb\_sc} \\ z_{sc\_sa} & z_{sc\_sb} & z_{sc\_sc} \end{bmatrix} \quad (30)$$

Next, we define the fault and sheath-to-ground impedance matrices. The core-to-sheath fault matrix is:

$$Z_F = \begin{bmatrix} Z_{FA} & 0 & 0 \\ 0 & Z_{FB} & 0 \\ 0 & 0 & Z_{FC} \end{bmatrix} \quad (31)$$

Fig. 2 shows the circuit diagram to model the core-to-ground faults using three phase-to-neutral impedances and one neutral-to-ground impedance. The modeling of the sheath-to-ground fault and the modeling of the sending- and receiving-end sheath grounding are similar to the circuit in Fig. 2.
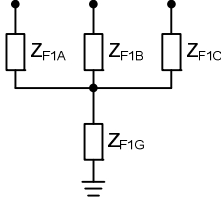

Fig. 2. Core-to-ground fault modeling circuit

The core-to-ground fault matrix is:

$$Z_{F1} = \begin{bmatrix} Z_{F1A} + Z_{F1G} & Z_{F1G} & Z_{F1G} \\ Z_{F1G} & Z_{F1B} + Z_{F1G} & Z_{F1G} \\ Z_{F1G} & Z_{F1G} & Z_{F1C} + Z_{F1G} \end{bmatrix} \quad (32)$$

The sheath-to-ground fault matrix is:

$$Z_{F2} = \begin{bmatrix} Z_{F2A} + Z_{F2G} & Z_{F2G} & Z_{F2G} \\ Z_{F2G} & Z_{F2B} + Z_{F2G} & Z_{F2G} \\ Z_{F2G} & Z_{F2G} & Z_{F2C} + Z_{F2G} \end{bmatrix} \quad (33)$$

The sending-end sheath-to-ground impedance matrix is:

$$Z_{SSG} = \begin{bmatrix} Z_{SSA} + Z_{SSG} & Z_{SSG} & Z_{SSG} \\ Z_{SSG} & Z_{SSB} + Z_{SSG} & Z_{SSG} \\ Z_{SSG} & Z_{SSG} & Z_{SSC} + Z_{SSG} \end{bmatrix} \quad (34)$$

The receiving-end sheath-to-ground impedance matrix is:

$$Z_{SRG} = \begin{bmatrix} Z_{SRA} + Z_{SRG} & Z_{SRG} & Z_{SRG} \\ Z_{SRG} & Z_{SRB} + Z_{SRG} & Z_{SRG} \\ Z_{SRG} & Z_{SRG} & Z_{SRC} + Z_{SRG} \end{bmatrix} \quad (35)$$

Next, we compute the source impedance matrices (36) and (37) in the phase frame of reference using the respective sending- and receiving-end positive- and zero-sequence source impedances.

$$Z_S = \left[Z(Z_{0S}, Z_{1S}) \right] \quad (36)$$

$$Z_R = \left[Z(Z_{0R}, Z_{1R}) \right] \quad (37)$$

where:

$$\left[Z(Z_0, Z_1) \right] = \begin{bmatrix} z_s(z_0, z_1) & z_m(z_0, z_1) & z_m(z_0, z_1) \\ z_m(z_0, z_1) & z_s(z_0, z_1) & z_m(z_0, z_1) \\ z_m(z_0, z_1) & z_m(z_0, z_1) & z_s(z_0, z_1) \end{bmatrix} \quad (38)$$

$$z_s(z_0, z_1) = \frac{2Z_1 + Z_0}{3} \quad (39)$$

$$z_m(z_0, z_1) = \frac{Z_0 - Z_1}{3} \quad (40)$$

D. Calculation of Underground Cable Core and Sheath Currents and Voltages at the Fault Point

To calculate the underground cable core and sheath currents and fault point voltages, we solve for Vector X in (7), as shown in (41).

$$X = A^{-1}E \quad (41)$$

Next, we calculate the sending- and receiving-end voltages using (42) and (43).

$$V_S = E_S - Z_S I_{S1} \quad (42)$$

$$V_R = E_R - Z_R I_{R1} \quad (43)$$

The bus voltages in (42) and (43) and the currents into the cable in Vector X from (41) are the quantities needed to calculate the apparent impedance (Z_{G_S}) seen by the distance element.

Next, we use (44) to calculate the a-phase ground distance loop impedance as seen from the sending Bus S.

$$Z_{G\_S} = \frac{V_{S\_a}}{I_{S\_ca} + k_0 I_{S\_res}} \quad (44)$$

where:

$$I_{S\_res} = I_{S\_ca} + I_{S\_cb} + I_{S\_cc} \quad (45)$$

k_0 is the zero-sequence current compensation factor used in ground distance relays, traditionally calculated from the assumption of homogeneous zero- and positive-sequence impedance.

E. Example: 1,200 Cu, 3,000-Meter, 230 kV Underground Cable Core-to-Sheath Fault

First, we compute the intermediate fault currents and voltages in a 3,000-meter, 230 kV single-core three-phase cable with its sheaths grounded at both cable ends through a resistance of 0.0001 Ω. The a-phase faults are placed at 300-meter intervals along the cable length, starting with $m = 0$ per unit (pu) at Bus S and $m = 1$ pu at Bus R. We ignore the cable shunt capacitances in this example to simplify the calculations.

The cable impedance matrix for this example is shown in the appendix. The sending- and receiving-end source impedances are:

- $Z_{1S} = j10\,\Omega$
- $Z_{0S} = j10\,\Omega$
- $Z_{1R} = j12\,\Omega$
- $Z_{0R} = j40\,\Omega$

Table I shows the magnitude of the sending-end terminal fault currents for faults at 300-meter intervals along the cable. I_{S_ca}, I_{S_cb}, and I_{S_cc} are the sending-end core currents in kA.

Similarly, Table II shows the magnitude of the sending-end terminal fault voltages V_{S_ca}, V_{S_cb}, and V_{S_cc} in kV.

TABLE I
FAULT CURRENT MAGNITUDES IN kA FROM BUS S

m in pu	$|I_{S\_ca}|$	$|I_{S\_cb}|$	$|I_{S\_cc}|$
0.0	13.729	1.725	1.725
0.1	13.273	1.728	1.728
0.2	13.266	1.731	1.731
0.3	13.258	1.734	1.734
0.4	13.250	1.737	1.737
0.5	13.241	1.740	1.740
0.6	13.231	1.743	1.743
0.7	13.222	1.745	1.745
0.8	13.211	1.748	1.748
0.9	13.200	1.751	1.751
1.0	13.189	1.753	1.753

TABLE II
FAULT VOLTAGE MAGNITUDES IN kV AT BUS S

m in pu	$|V_{S\_ca}|$	$|V_{S\_cb}|$	$|V_{S\_cc}|$
0.0	0.001	142.269	142.136
0.1	0.316	142.324	142.117
0.2	0.620	142.378	142.099
0.3	0.913	142.430	142.082
0.4	1.195	142.480	142.066
0.5	1.466	142.528	142.050
0.6	1.727	142.575	142.930
0.7	1.977	143.619	143.021
0.8	2.216	143.662	143.008
0.9	2.446	142.704	141.996
1.0	2.665	142.743	141.985

Fig. 3 and Fig. 4 show the a-phase ground loop impedances for the sending and receiving ends, respectively, using (44) and (45) with a ground zero-sequence current compensation factor of $k_0 = 0.66/\underline{0}^0$.
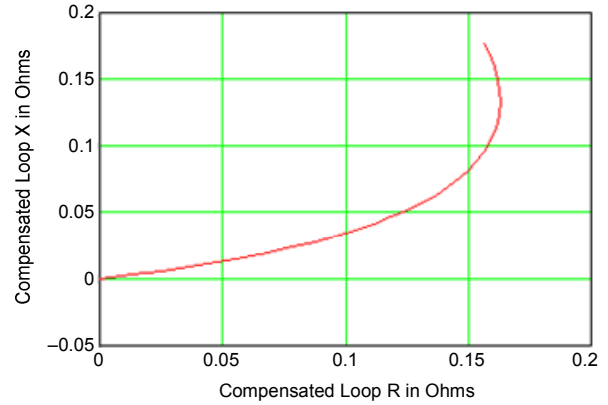

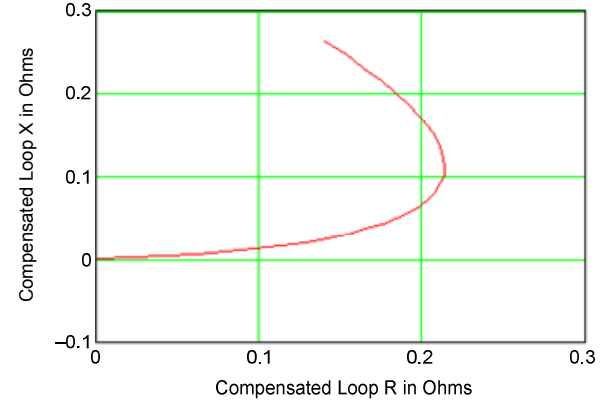Fig. 3.   Bus S ground loop impedance ($k_0 = 0.66/\underline{0}^0$) in ohms

Fig. 4.   Bus R ground loop impedance ($k_0 = 0.66/\underline{0}^0$) in ohms

Fig. 5 and Fig. 6 show the sending-end (Bus S) ground loop resistance and reactance ($k_0 = 0.66/\underline{0}^0$). Note that faults are applied every 300 meters, with $m = 0$ at the sending end and $m = 1$ at the receiving end.
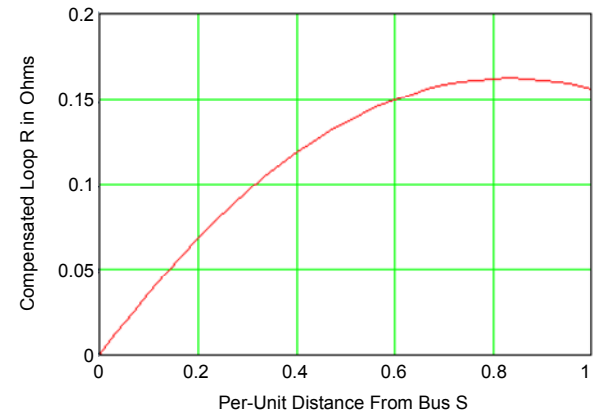

Fig. 5.   Bus S ground loop resistance ($k_0 = 0.66/\underline{0}^0$) in ohms
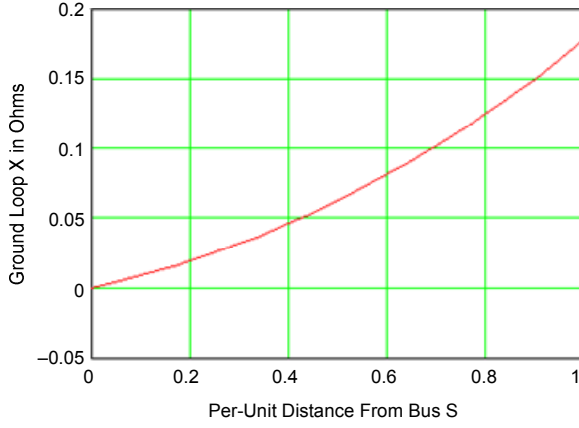
Fig. 6.   Bus S ground loop reactance ($k_0 = 0.66/\underline{0}^0$) in ohms

Fig. 7.   Major underground cable section consisting of three minor sections

III. INTERMEDIATE FAULTS IN CROSS-BONDED UNDERGROUND CABLES WITH SHEATHS GROUNDED AT BOTH CABLE ENDS

Underground cable sheath grounding and bonding are applied to perform the following:

- Reduce sheath losses to a minimum.
- Limit sheath voltages as required by sheath sectionalizing joints.
- Maintain a continuous sheath circuit for fault current return and adequate lightning and switching surge protection.

The most common sheath bonding methods are single-point bonding, solid bonding, and cross-bonding [3].

Cross-bonding of single-conductor cables attempts to neutralize the total induced voltage in the cable sheaths to minimize the circulating current and losses in the cable sheaths while permitting increased cable spacing, increased current-carrying capacity, and longer runs of cable lengths.

The most basic form of cross-bonding consists of sectionalizing the cable into three minor sections of equal length and cross connecting the sheaths at each minor section. Longer cable circuits may consist of a number of major sections in series. When the number of minor sections is divisible by three, the cable circuit can be arranged to consist of more than one major section. Three minor cable sections form a major section. The cable cores may be transposed at each minor section, and the sheaths are cross-bonded at the end of each minor section throughout the whole cable route. The three cable sheaths are bonded and grounded at the two ends of the route only. Fig. 7 shows a major section of an underground cable having three minor sections.

We use the cable model in Fig. 7 to study faults along the cable. The cable cores in Fig. 7 are not transposed. The cable sheaths are transposed at each minor section and grounded at the beginning and end of the major section.
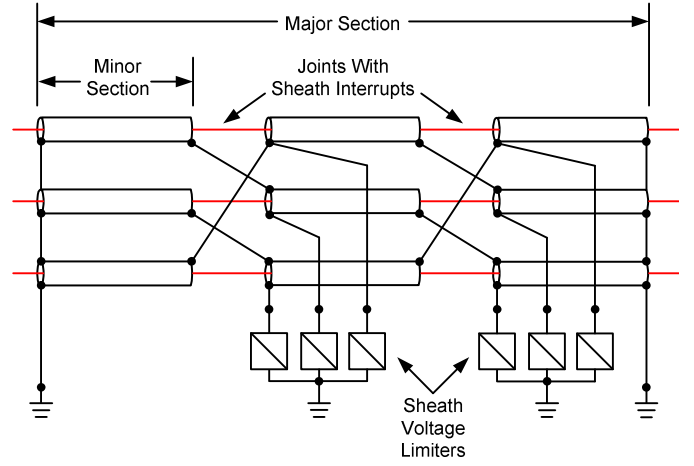
A. Conductor Transpositions

The phase conductors in a three-phase circuit are mutually coupled, and the currents in any one conductor produce voltage drops in adjacent conductors. These induced voltage drops can be unequal even for balanced conductor currents because the mutual impedances depend entirely on the physical arrangement of the phase conductors. One way to equalize the mutual inductances is to transpose the line conductors so that each conductor occupies the next physical position for one-third of the total line length in a regular sequence, such as *a-b-c*, *b-c-a*, and *c-a-b*.

Mathematically, we can use a rotation matrix to take care of the effect of conductor transpositions [4]. Fig. 8 shows a forward transposition (clockwise) of three phase conductors, where Conductor a in Section One (S-I) occupies Position 2 in Section Two (S-II), Conductor b in S-I occupies Position 3 in S-II, and Conductor c in S-I occupies Position 1 in S-II.
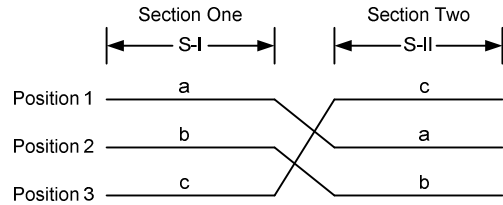

Fig. 8.   Forward conductor transposition

Equation (46) is the rotation matrix corresponding to the forward transposition in Fig. 8.

$$R = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \tag{46}$$

Premultiplying an impedance matrix by the rotation matrix (46) has the effect of rotating the self- and mutual impedances of a conductor in Position 1 to the matrix positions initially occupied by the self- and mutual impedances of a conductor in Position 2. In addition, it rotates the self- and mutual impedances of a conductor in Position 2 to the matrix positions initially occupied by the self- and mutual impedances of a conductor in Position 3 and those of a

conductor in Position 3 to the ones occupied previously by a conductor in Position 1.

The rotation matrix has a number of interesting properties. First, the inverse of the rotation matrix exists and is equal to its transpose. Note that this is a counterclockwise, or reverse, rotation because it rearranges the matrix elements in exactly the reverse of the reordering produced by the forward rotation matrix.

$$R^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} = R^T \tag{47}$$

The result of postmultiplying a matrix by R^{-1} is to move the first matrix column to the second position, the second column to the third position, and the third column to the first position.

Note that two rotations produce the same result as one rotation in the opposite direction.

$$R^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \tag{48}$$

If the conductors are not transposed from one section to the next, the rotation matrix is equal to the unit or identity matrix. In addition, three forward rotations result in the unit matrix:

$$R^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{49}$$

The voltage drop per-unit length in S-II as a function of the phase impedance matrix per-unit length of S-I is given by (50).

$$\begin{bmatrix} Va \\ Vb \\ Vc \end{bmatrix}_{S\text{-}II} = R^T Z_{PH\_(S\text{-}I)} R \begin{bmatrix} Ia \\ Ib \\ Ic \end{bmatrix}_{S\text{-}II} \tag{50}$$

The rotation matrix for a three-phase underground single-core cable with sheaths (a total of six conductors) is a 6x6 matrix. Its elements depend on whether both the core and sheaths are transposed at each minor section, whether only the sheaths are transposed, and whether the rotation is forward or reverse. The rotation matrix for the underground cable shown in Fig. 7 is:

$$R = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{51}$$

Note that in Fig. 7, the cable cores are not transposed from one minor section to the next. That is why the upper left-hand side 3x3 submatrix of (51) is equal to the unit matrix. Also note that the first transposition in Fig. 7 is in the forward, or clockwise, direction, similar to the one shown in Fig. 8. This is the reason why the lower right-hand side 3x3 submatrix is equal to the forward rotation matrix of (46).

B. Calculating Intermediate Faults in S-I

To calculate faults in S-I, we take advantage of what we discussed in Section II of this paper on how to formulate Matrix A with some additional modifications to take into consideration cable minor sections S-II and S-III and their sheath transpositions. We also introduce a slightly modified notation for the cable self- and mutual impedances to distinguish between the three cable minor sections. For example, Z_{CC_1} is the core-to-core self- and mutual impedances for minor section S-I. The subscript 1 indicates impedances of S-I, 2 of S-II, and 3 of Section Three (S-III), respectively. Likewise, Z_{SS_1} is the sheath-to-sheath self- and mutual impedances for minor section S-I, and Z_{CS_1} is the core-to-sheath mutual impedances of S-I. We use the cable system in Fig. 9 to study faults in S-I of the cable.
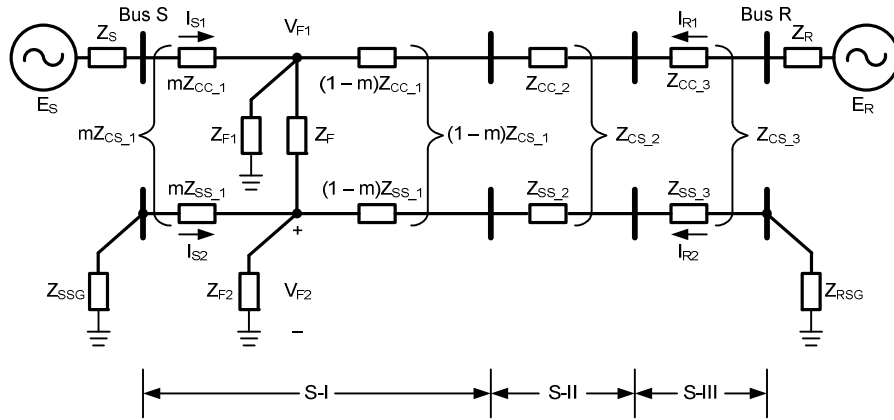

Fig. 9.   Faults in S-I of underground cable

$$A = \begin{bmatrix} (Z_S + mZ_{CC\_1}) & mZ_{CS\_1} & 0 & 0 & I & 0 \\ 0 & 0 & \left[Z_{SR} + (1-m)Z_{CC\_1} + Z_{CC\_2} + Z_{CC\_3}\right] & \left[(1-m)Z_{CC\_1} + Z_{CC\_2} + Z_{CC\_3}\right] & I & 0 \\ mZ_{CS\_1}^T & (Z_{SSG} + mZ_{SS\_1}) & 0 & 0 & 0 & I \\ 0 & 0 & \left[(1-m)Z_{CS\_1}^T + Z_{CS\_2}^T + Z_{CS\_3}^T\right] & \left[Z_{RSG} + (1-m)Z_{CS\_1}^T + Z_{CS\_2}^T + Z_{CS\_3}^T\right] & 0 & I \\ I & 0 & I & 0 & -\left(Z_{F1}^{-1} + Z_F^{-1}\right) & Z_F^{-1} \\ 0 & I & 0 & I & Z_F^{-1} & -\left(Z_{F2}^{-1} + Z_F^{-1}\right) \end{bmatrix} \quad (52)$$

Matrix A for faults in S-I is shown in (52).

The S-II and S-III submatrices are derived from the S-II and S-III cable self- and mutual impedances, taking into consideration the transpositions, as shown in (53) through (56).

R is the rotation matrix in (51), and Z_C is the cable self- and mutual impedance matrix in (26). Note that the transposition from S-II to S-I of the cable is in the reverse direction, which is reflected in (53).

$$Z_{C\_2} = \frac{1}{3} R Z_C R^T \quad (53)$$

$$Z_{C\_2} = \begin{bmatrix} Z_{CC\_2} & Z_{CS\_2} \\ Z_{SC\_2} & Z_{SS\_2} \end{bmatrix} = \begin{bmatrix} Z_{CC\_2} & Z_{CS\_2} \\ Z_{CS\_2}^T & Z_{SS\_2} \end{bmatrix} \quad (54)$$

Also note that there are two reverse transpositions from S-III toward S-I of the cable (equivalent to a forward rotation), which is reflected in (55).

$$Z_{C\_3} = \frac{1}{3} R^T Z_C R \quad (55)$$

$$Z_{C\_3} = \begin{bmatrix} Z_{CC\_3} & Z_{CS\_3} \\ Z_{SC\_3} & Z_{SS\_3} \end{bmatrix} = \begin{bmatrix} Z_{CC\_3} & Z_{CS\_3} \\ Z_{CS\_3}^T & Z_{SS\_3} \end{bmatrix} \quad (56)$$

Table III shows the fault currents in kA at Bus S for faults in S-I of the example in Section II, Subsection E, where each minor section is 1,000 meters and the sheaths are cross-bonded at each minor section and grounded at both cable ends.

TABLE III
FAULT CURRENT MAGNITUDES IN KA FROM BUS S

m in pu of S-I	$|I_{S\_ca}|$	$|I_{S\_cb}|$	$|I_{S\_cc}|$
0.0	13.279	1.721	1.721
0.1	13.269	1.721	1.721
0.2	13.259	1.722	1.722
0.3	13.248	1.722	1.722
0.4	13.236	1.723	1.723
0.5	13.224	1.723	1.723
0.6	13.211	1.723	1.723
0.7	13.198	1.723	1.723
0.8	13.184	1.723	1.723
0.9	13.170	1.723	1.723
1.0	13.155	1.723	1.723

Table IV shows the voltages in kV at Bus S for faults in S-I.

TABLE IV
FAULT VOLTAGE MAGNITUDES IN KV AT BUS S

m in pu of S-I	$|V_{S\_ca}|$	$|V_{S\_cb}|$	$|V_{S\_cc}|$
0.0	0.001	142.154	142.202
0.1	0.336	142.202	142.160
0.2	0.662	142.249	142.120
0.3	0.976	142.293	142.080
0.4	1.181	142.336	142.041
0.5	1.575	142.376	142.003
0.6	1.859	142.415	142.966
0.7	2.133	143.452	143.930
0.8	2.398	143.487	143.894
0.9	2.654	142.521	141.860
1.0	2.900	142.552	141.827

C. Calculating Intermediate Faults in S-II

To calculate intermediate faults in S-II of the underground cable, we first calculate the proper cable impedances for S-III and S-I, taking into consideration the sheath transpositions, using (57) through (60).

R is the rotation matrix in (51), and Z_C is the cable self- and mutual impedance matrix in (26). Note that the sheath transposition from S-III to S-II of the cable is in the reverse direction, which is reflected in (57).

$$Z_{C\_3} = \frac{1}{3} R Z_C R^T \quad (57)$$

$$Z_{C\_3} = \begin{bmatrix} Z_{CC\_3} & Z_{CS\_3} \\ Z_{SC\_3} & Z_{SS\_3} \end{bmatrix} = \begin{bmatrix} Z_{CC\_3} & Z_{CS\_3} \\ Z_{CS\_3}^T & Z_{SS\_3} \end{bmatrix} \quad (58)$$

Note that the transposition from S-I to S-II of the cable is in the forward direction, which is reflected in (59).

$$Z_{C\_1} = \frac{1}{3} R^T Z_C R \quad (59)$$

$$Z_{C\_1} = \begin{bmatrix} Z_{CC\_1} & Z_{CS\_1} \\ Z_{SC\_1} & Z_{SS\_1} \end{bmatrix} = \begin{bmatrix} Z_{CC\_1} & Z_{CS\_1} \\ Z_{CS\_1}^T & Z_{SS\_1} \end{bmatrix} \quad (60)$$

$$A = \begin{bmatrix}
\left(Z_S + Z_{CC\_1} + mZ_{CC\_2}\right) & Z_{CS\_1} + mZ_{CS\_2} & O & O & I & O \\
O & O & \left[Z_{SR} + (1-m)Z_{CC\_2} + Z_{CC\_3}\right] & \left[(1-m)Z_{CS\_2} + Z_{CS\_3}\right] & I & O \\
Z_{CS\_1}^T + mZ_{CS\_2}^T & \left(Z_{SSG} + Z_{SS\_1} + mZ_{SS\_2}\right) & O & O & O & I \\
O & O & \left[(1-m)Z_{CS\_2}^T + Z_{CS\_3}^T\right] & \left[Z_{RSG} + (1-m)Z_{SS\_2} + Z_{SS\_3}\right] & O & I \\
I & O & I & O & -\left(Z_{F1}^{-1} + Z_F^{-1}\right) & Z_F^{-1} \\
O & I & 0 & I & Z_F^{-1} & -\left(Z_{F2}^{-1} + Z_F^{-1}\right)
\end{bmatrix} \quad (61)$$

Next, we formulate Matrix A using (61).

The formulation of Matrices E and X is the same as discussed in Section II, Subsections A and B. To calculate the voltages and currents at Buses S and R, we use the same approach discussed in Section II, Subsection D.

Table V and Table VI show the fault currents in kA and fault voltages in kV at Bus S for a-phase core-to-sheath faults in S-II.

TABLE V
FAULT CURRENT MAGNITUDES IN KA FROM BUS S

m in pu of S-II	$|I_{S\_ca}|$	$|I_{S\_cb}|$	$|I_{S\_cc}|$
0.0	13.044	1.773	1.716
0.1	13.036	1.774	1.717
0.2	13.028	1.775	1.718
0.3	13.019	1.775	1.718
0.4	13.009	1.776	1.719
0.5	13.999	1.776	1.720
0.6	13.989	1.777	1.720
0.7	13.978	1.777	1.721
0.8	13.966	1.778	1.721
0.9	13.954	1.778	1.721
1.0	13.942	1.778	1.721

TABLE VI
FAULT VOLTAGE MAGNITUDES IN KV AT BUS S

m in pu of S-II	$|V_{S\_ca}|$	$|V_{S\_cb}|$	$|V_{S\_cc}|$
0.0	3.505	142.852	141.787
0.1	3.678	142.885	141.763
0.2	3.846	142.916	141.741
0.3	4.007	142.945	141.719
0.4	4.163	142.973	141.698
0.5	4.312	142.998	141.678
0.6	4.454	143.022	141.659
0.7	4.591	143.044	141.641
0.8	4.721	143.064	141.624
0.9	4.846	143.083	141.607
1.0	4.965	143.099	141.592

D. Calculating Intermediate Faults in S-III

To calculate intermediate faults in S-III of the cable, we first calculate the proper cable impedances for S-I and S-II, taking into consideration the sheath transpositions, using (62) through (65).

R is the rotation matrix in (51), and Z_C is the cable self- and mutual impedance matrix in (26). Note that there are two forward transpositions from S-I toward S-III of the cable (equivalent to a reverse rotation), which is reflected in (62).

$$Z_{C\_1} = \frac{1}{3}RZ_CR^T \quad (62)$$

$$Z_{C\_1} = \begin{bmatrix} Z_{CC\_1} & Z_{CS\_1} \\ Z_{SC\_1} & Z_{SS\_1} \end{bmatrix} = \begin{bmatrix} Z_{CC\_1} & Z_{CS\_1} \\ Z_{CS\_1}^T & Z_{SS\_1} \end{bmatrix} \quad (63)$$

Note that the transposition from S-II to S-III of the cable is in the forward direction, which is reflected in (64).

$$Z_{C\_2} = \frac{1}{3}R^TZ_CR \quad (64)$$

$$Z_{C\_2} = \begin{bmatrix} Z_{CC\_2} & Z_{CS\_2} \\ Z_{SC\_2} & Z_{SS\_2} \end{bmatrix} = \begin{bmatrix} Z_{CC\_2} & Z_{CS\_2} \\ Z_{CS\_2}^T & Z_{SS\_2} \end{bmatrix} \quad (65)$$

Table VII and Table VIII show the fault currents in kA and fault voltages in kV at Bus S for a-phase core-to-sheath faults in S-III.

TABLE VII
FAULT CURRENT MAGNITUDES IN KA FROM BUS S

m in pu of S-III	$|I_{S\_ca}|$	$|I_{S\_cb}|$	$|I_{S\_cc}|$
0.0	12.916	1.782	1.782
0.1	12.910	1.784	1.784
0.2	12.904	1.785	1.785
0.3	12.897	1.785	1.785
0.4	12.890	1.787	1.787
0.5	12.883	1.787	1.787
0.6	12.874	1.788	1.788
0.7	12.866	1.789	1.789
0.8	12.856	1.790	1.790
0.9	12.846	1.790	1.790
1.0	12.836	1.791	1.791

TABLE VIII
FAULT VOLTAGE MAGNITUDES IN KV AT BUS S

m in pu of S-III	$|V_{S\_ca}|$	$|V_{S\_cb}|$	$|V_{S\_cc}|$
0.0	5.146	143.127	141.939
0.1	5.193	143.145	141.933
0.2	5.235	143.161	141.928
0.3	5.271	143.176	141.925
0.4	5.305	143.189	141.922
0.5	5.334	143.200	141.921
0.6	5.360	143.210	141.921
0.7	5.384	143.217	141.921
0.8	5.406	143.223	141.923
0.9	5.428	143.227	141.926
1.0	5.449	143.229	141.929

E. Comparison of Mathcad and EMTP Results

The results in Table III and Table IV shows the voltages in kV at Bus S for faults in S-I.

Table IV were calculated using Mathcad. The underground cable in Fig. 7 was modeled in the phase frame of reference using the equations presented in Section II, Subsections A through D and in Section III, Subsection B. The cable model consists of three minor sections (1,000 meters each). The sheaths are transposed forward at each minor section and grounded at each cable end with 0.0001 Ω resistances.

To verify the results of the proposed method, an EMTP model of the same cable was developed. The cable was modeled in 100-meter increments using mutual R-L elements. The cable shunt capacitances were not modeled so far in this paper in order to simplify the calculations and for ease of understanding the proposed method.

Table IX and Table X show the comparison of the Bus S current and voltages for faults at two locations in S-I, at 20 and 50 percent from Bus S.

TABLE IX
MATHCAD AND EMTP RESULTS FOR FAULTS IN S-I
WITH THE FAULT POINT 20 PERCENT FROM BUS S

		Mathcad		EMTP	
		Mag	Angle	Mag	Angle
Core Currents at Bus S in kA	I_{S_ca}	13.259	−89.73	13.259	−89.73
	I_{S_cb}	1.722	−89.74	1.722	−89.74
	I_{S_cc}	1.722	−89.74	1.722	−89.74
Core Voltages at Bus S in kV	V_{S_ca}	0.662	−72.11	0.662	−72.11
	V_{S_cb}	142.249	−126.01	142.249	−126.01
	V_{S_cc}	142.220	126.04	142.220	126.04

TABLE X
MATHCAD AND EMTP RESULTS FOR FAULTS IN S-I
WITH THE FAULT POINT 50 PERCENT FROM BUS S

		Mathcad		EMTP	
		Mag	Angle	Mag	Angle
Core Currents at Bus S in kA	I_{S_ca}	13.224	−89.36	13.224	−89.36
	I_{S_cb}	1.723	−89.23	1.723	−89.23
	I_{S_cc}	1.723	−89.23	1.723	−89.23
Core Voltages at Bus S in kV	V_{S_ca}	1.575	−69.29	1.575	−69.285
	V_{S_cb}	142.376	−125.97	142.376	−125.97
	V_{S_cc}	142.003	126.08	142.003	126.08

Note that the EMTP steady-state results are the same as the results produced by the proposed method using the phase frame of reference approach and modeling the cross-bonded cable in Mathcad.

Note also that the EMTP and Mathcad results are exactly the same for faults in S-II and S-III of the cable. The results in S-II and S-III are not shown here in the interest of space.

F. Ground Loop Impedances From Bus S

To calculate the a-phase ground loop impedances as seen from Bus S, we use (44) and (45). The ground current compensation factor was set to $k_0 = 0.66/\underline{0}^0$, which is typical of a transmission line where Z_{0L} is three times Z_{1L} for illustration purposes. Fig. 10 shows the compensated loop resistance, and Fig. 11 shows the compensated loop reactance in ohms as seen from Bus S.
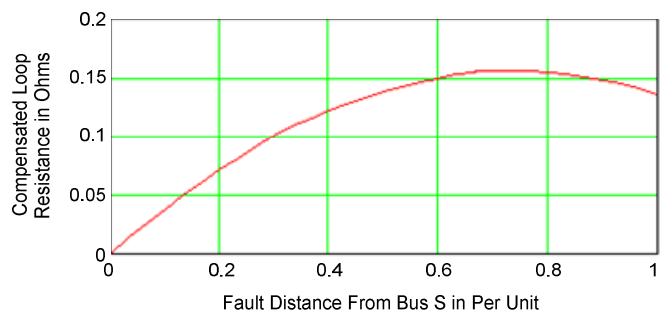

Fig. 10. Compensated loop resistance ($k_0 = 0.66/\underline{0}^0$) in ohms for cross-bonded cable

Fig. 11. Compensated loop reactance ($k_0 = 0.66/\underline{0}^0$) in ohms for cross-bonded cable

Fig. 12 shows the compensated loop impedance in ohms as seen from Bus S.
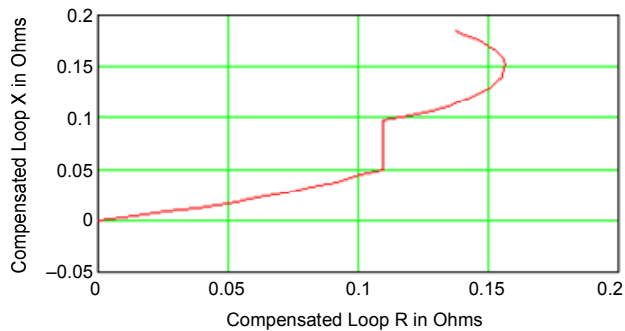

Fig. 12.   Compensated loop impedance ($k_0 = 0.66/\underline{0}^0$) in ohms for cross-bonded cable

Note that in cross-bonded cables where the sheaths are transposed at each minor section and grounded at both cable ends, the compensated loop resistance is not the maximum for a fault at the remote end.

Note also that moving the fault from the end of a minor section to the beginning of the next minor section causes a different return path for the ground fault current and, consequently, causes a discontinuity in the compensated loop impedance. This discontinuity, shown in Fig. 11 and Fig. 12, offers some advantages in obtaining selectivity for a Zone 1 setting distance element for faults in the last minor section. Note that the discontinuity is more pronounced when the fault is moved from the first to the second minor section.

G. Considerations for Setting Ground Distance Elements

Ground distance elements measure fault impedance in terms of positive-sequence impedance only. Set the zero-sequence current compensation factor so that the Zone 1 ground distance elements do not see faults external to the protected cable, while the Zone 2 and Zone 3 ground distance elements see all cable internal faults and coordinate with distance relays on adjacent line or cable circuits.

The choice of a zero-sequence current compensation factor can influence the reach and performance of ground distance relays. Choose a zero-sequence current compensation factor that obtains a constant or increasing slope of the compensated loop reactance for faults at the end of the cable. Do this by choosing a complex zero-sequence current compensation factor corresponding to the cable under study or by selecting a fictitious scalar ground zero-sequence current compensation factor that would compensate correctly for faults at the end of the cable.

Consider other parameters in addition to the different behavior of the compensated loop impedance, depending on sheath bonding and grounding methods. Network topology plays an important role in selecting settings for underground cable applications. In some applications, parallel cables are installed between two substations. In others, there are mixed overhead and underground sections. Also consider adjacent line sections, whether cables or overhead lines.

For example, in the case of parallel cables, select the proper zero-sequence current compensation factor for Zone 1 by placing a phase-to-ground fault at the remote terminal with the parallel cable out of service. Find the ground distance reactance measurement that does not overreach for that fault using the two zero-sequence current compensation factors that correspond to two different return paths: sheath return only and sheath and ground return. Use all three different cable zero-sequence impedances (see the appendix) in the fault study. Select the zero-sequence compensation factor that does not provide any overreach for the sheath return alone or for the sheath and ground return path.

For the overreaching zones, select the zero-sequence compensation factor so that the ground distance overreaching zones do not underreach for any internal ground faults. Select the zero-sequence current compensation factor that corresponds to the zero-sequence impedance of the cable with ground return only. Place both parallel cables in service, simulate a line-to-ground fault at the remote terminal, and calculate the ground distance reactance measurement for each of the three possible zero-sequence cable impedances.

Modern digital ground distance relay elements offer the user more options in achieving better performance of ground distance element measurement than do their older electromechanical and static counterparts. Modern elements offer more than one complex zero-sequence current compensation factor, with a wide range of magnitude and angle settings. In general, negative-sequence current polarizing is the preferred choice for cable applications because the negative-sequence network is more homogeneous than the zero-sequence network. In addition, modern digital relays offer a nonhomogeneous correction angle setting to help prevent overreach or underreach for ground faults at a specific fault point by compensating the angle of the reactance line.

Although most of this discussion was on the ground distance element, phase distance elements can also be affected by large capacitive charging currents. The large charging currents can result in an overreaching effect of a Zone 1 phase distance relay.

Protecting underground cables with distance relays can be quite challenging and difficult to achieve because of the cable electrical characteristics, the influence of grounding methods and return currents in the zero-sequence impedance of the cable, the nonlinear behavior of the compensated loop impedance, and the short cable length in many applications. For all of these reasons and the complexities involved in making the proper settings, most users prefer to protect high-voltage underground cables using line current differential relays. Distance relays are typically applied in a directional comparison blocking or unblocking scheme and for backup protection.

Modern digital relays have integrated into one relay box a complete line differential relaying scheme with full distance protection elements, including communications-assisted protection logic, negative- and zero-sequence directional elements, and a plethora of other overcurrent elements. With modern digital relays, we now have a choice of many different relay elements for the protection of underground cables, some

of which may be better suited than others. Supplementing ground distance elements with negative-sequence directional elements in a communications-assisted tripping scheme provides excellent resistive coverage for high-resistance ground faults (for example, during a flashover of a contaminated pothead). The use of negative-sequence directional elements has also been successful in a directional comparison scheme for the protection of submarine cables [5].

IV. CABLE SHUNT CAPACITANCE CONSIDERATIONS

The electrical characteristics of underground cables differ significantly from overhead transmission lines. Underground cables exhibit a much lower series inductance and a much higher shunt capacitance.

The series inductance of cable circuits is typically 30 to 50 percent lower than that of overhead lines because of the close spacing of cable conductors. The difference in the cable shunt capacitance is even more pronounced and can be 30 to 40 times higher than that of overhead lines. The closer proximity of the cable conductor to ground potential, surrounded by the cable grounded sheath, and the dielectric constant of the insulation, which is several times that of air, cause this difference.

Up to this point in this paper, we have ignored the cable shunt capacitance so as to not complicate the intermediate fault calculations. However, for long cables, it is important to take into consideration the cable shunt capacitance in the analysis of intermediate fault calculations.

For short cables consisting of one section (of about 1,000 meters) or for cross-bonded cables with three minor sections (300 meters each), we can include the shunt cable capacitances in a similar way to how we handled shunt core-to-ground faults. Note that we must represent half of the cable capacitance at each cable end. In addition, we must modify Matrix A accordingly to take care of the additional shunt branches that are now part of the cable model.

For very long cables having too many major sections, the proposed method becomes more complex and the user is advised to use other programs, such as EMTP, to study intermediate faults. EMTP is the preferred approach; however, other methods are discussed in [6] and [7], where the system multiphase admittance matrix is formed to study three-phase unbalanced networks.

V. CONCLUSION

This paper illustrates the analysis of intermediate faults in underground ac cables using the phase frame of reference and Mathcad, an engineering calculation software package.

The complexity of fault calculations in underground cables with sheath transpositions and sheath grounding necessitates the use of a multiphase analysis method instead of using the symmetrical component method.

Numerical examples illustrate the use of the proposed method for a cable with its sheaths grounded at both cable ends.

A second numerical example illustrates the results of intermediate fault calculations in each minor section of a cross-bonded cable, consisting of one major section with its sheaths transposed at each minor section and grounded at both cable ends.

The results of the proposed method using Mathcad and the results from an EMTP analysis for faults along the cable are identical, validating the proposed method as an alternate method to using a steady-state EMTP solution.

The zero-sequence impedance of the cable is not linearly related to fault distance and is affected by cable bonding and grounding methods.

The application of ground distance elements for cable protection is complex, and protection engineers should be careful when making ground distance element settings, including proper selection of the zero-sequence current compensation factor. This paper presents considerations for setting ground distance elements and the zero-sequence current compensation factor.

VI. APPENDIX

The single-conductor cable data used throughout the paper are the following:
- Cable type: 230 kV 1,200 mm^2 copper
- Cable length: 1,000 m
- Conductor radius: 2.15 E-02 m
- Insulation radius: 4.52 E-02 m
- Sheath radius: 4.98 E-02 m
- Polyvinyl chloride (PVC) radius: 5.38 E-02 m
- Conductor resistivity: 1.72 E-08 Ωm at 20°C
- Sheath resistivity: 2.14 E-07 Ωm at 20°C
- Permittivity of insulation: 1.0
- Permittivity of PVC: 8.0
- Earth resistivity: 100.0 Ωm

Fig. 13 shows the underground cable conductors laid in a trefoil configuration. Note that the cable sheaths are grounded at both cable ends, and there is no ground continuity conductor.
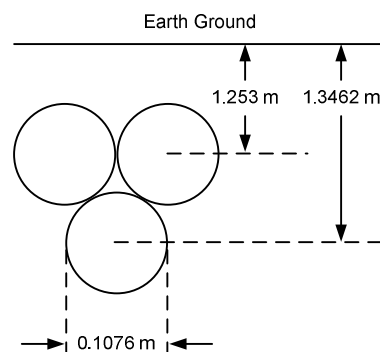
Fig. 13.   230 kV cable trefoil laying arrangement

$$Z_C = \begin{pmatrix} 0.231074+2.437653j & 0.177067+2.030733j & 0.177087+2.030739j & 0.177137+2.215608j & 0.177067+2.030733j & 0.177087+2.030739j \\ 0.177067+2.030733j & 0.231033+2.437698j & 0.177067+2.030733j & 0.177067+2.030733j & 0.177096+2.215653j & 0.177067+2.030733j \\ 0.177087+2.030739j & 0.177067+2.030733j & 0.231074+2.437653j & 0.177087+2.030739j & 0.177067+2.030733j & 0.177137+2.215608j \\ 0.177137+2.215608j & 0.177067+2.030733j & 0.177087+2.030739j & 0.644715+2.211957j & 0.177067+2.030733j & 0.177087+2.030739j \\ 0.177067+2.030733j & 0.177096+2.215653j & 0.177067+2.030733j & 0.177067+2.030733j & 0.644673+2.211999j & 0.177067+2.030733j \\ 0.177087+2.030739j & 0.177067+2.030733j & 0.177137+2.215608j & 0.177087+2.030739j & 0.177067+2.030733j & 0.644715+2.211957j \end{pmatrix} \quad (66)$$

The calculated phase impedance matrix of the cable (3,000 meters long) using EMTP in ohms is shown in (66).

The cable sequence impedances are [1]:

- Cable positive-sequence Z_{1C} (Ω):
  $Z_{1C} = 0.039 + j\,0.127$
- Zero-sequence conductor Z_{0C} (Ω):
  $Z_{0C} = 0.195 + j\,2.166$
- Zero-sequence sheath Z_{0S} (Ω): $Z_{0S} = 0.333 + j\,2.091$
- Zero-sequence mutual Z_{0m} (Ω): $Z_{0m} = 0.177 + j\,2.092$

To calculate the zero-sequence impedance of the cable (Z_0) for the three different return paths, we can use the equivalent circuit shown in Fig. 14.
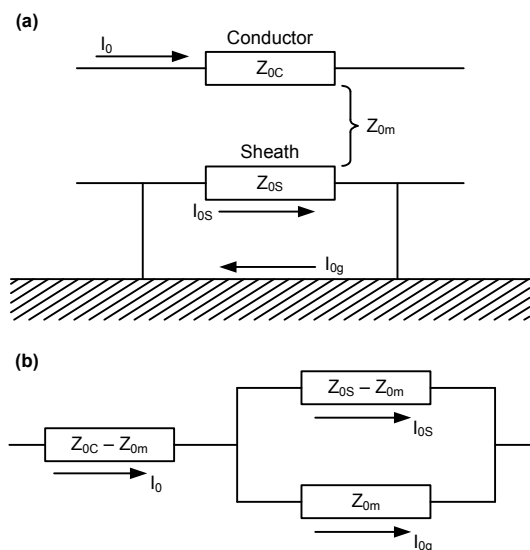

Fig. 14.   Zero-sequence return currents (a) and equivalent circuit (b)

The cable zero-sequence impedances for the three possible current return paths are:

- Current return in the sheath only:

$$Z_0 = Z_{0C} + Z_{0S} - 2 \cdot Z_{0m} \quad (67)$$

$$Z_0 = 0.174 + j0.073\,\Omega \quad (68)$$

- Current return in the ground only:

$$Z_0 = Z_{0C} - Z_{0m} + Z_{0m} = Z_{0C} \quad (69)$$

$$Z_0 = 0.195 + j2.166\,\Omega \quad (70)$$

- Current in the sheath and ground in parallel:

$$Z_0 = Z_{0C} - Z_{0m} + \frac{(Z_{0S} - Z_{0m}) \cdot Z_{0m}}{Z_{0S}} = Z_{0C} - \frac{Z_{0m}^2}{Z_{0S}} \quad (71)$$

$$Z_0 = 0.172 + j0.084\,\Omega \quad (72)$$

VII. REFERENCES

[1] D. Tziouvaras, "Protection of High-Voltage AC Cables," proceedings of the 32nd Annual Western Protective Relay Conference, Spokane, WA, October 2005.

[2] D. Tziouvaras, "Analysis of Complex Power System Faults and Operating Conditions," proceedings of the 35th Annual Western Protective Relay Conference, Spokane, WA, October 2008.

[3] IEEE Standard 575-1988, IEEE Guide for Application of Sheath-Bonding Methods for Single-Conductor Cables and the Calculation of Induced Voltages and Currents in Cable Sheaths.

[4] P. M. Anderson, *Analysis of Faulted Power Systems*. The Iowa State University Press, Ames, Iowa, 1973, pp. 84–90.

[5] J. Vargas, A. Guzmán, and J. Robles, "Underground/Submarine Cable Protection Using a Negative-Sequence Directional Comparison Scheme," proceedings of the 26th Annual Western Protective Relay Conference, Spokane, WA, October 1999.

[6] J. Arrillaga, C. P. Arnold, and B. J. Harker, *Computer Modelling of Electrical Power Systems*. John Wiley and Sons, 1983, pp. 10–42.

[7] R. Benato and A. Paolucci, *EHV AC Undergrounding Electrical Power*. Springer, 2010, pp. 119–137.

VIII. BIOGRAPHY

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